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Verification of the NIMSTELL code via application to linear magnetohydrodynamic instabilities in stellarators

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A new simulation tool for macroscale dynamics in stellarators, NIMSTELL [C. R. Sovinec, S. A. Patil, and J. V. Cauilan, UW-CPTC 25-1 (2025)], is verified through benchmarking with the time-dependent code JOEKE [N. Nikulsin, *et al.*, Phys. Plasmas **29**, 063901 (2022)] and the eigenvalue code CASTOR3D [E. Strumberger and S. Günter, Nucl. Fusion **57**, 016032 (2016)]. The comparisons establish NIMSTELL's accuracy and reliability in representative configurations, where analytical results are not tractable. Results from all three codes are assessed for linear tearing and ballooning instabilities in W7-A and tearing in a quasi-axisymmetric (QA) configuration. NIMSTELL and CASTOR3D are also applied to global non-resonant interchange in a quasi-helically symmetric (QH) configuration. The computed NIMSTELL and CASTOR3D growth rates agree to the level of 1.5% and 3% for tearing modes in W7-A and QA configurations, respectively, to 5.8% for a W7-A ballooning mode, and to 1.6% for a global interchange mode in QH configurations. While NIMSTELL results also agree with JOEKE for a majority of the benchmark cases, the numerical discrepancies are somewhat larger than those with respect to CASTOR3D in specific parameter regimes. The discrepancies are largely ascribed to the use of reduced-MHD modeling in the JOEKE computations. In addition to benchmarking, the nonlinear capability of NIMSTELL is demonstrated by modeling the loss of pressure due to global interchange modes in the QH configuration. The successful verification presented in this work provides support for applying NIMSTELL to physically relevant stability analyses of current experiments and future designs.

PACS numbers:

I. INTRODUCTION

Contemporary research on the stellarator magnetic confinement concept has emphasized optimization with respect to neoclassical and turbulent transport, confinement of energetic particles, magnetohydrodynamic (MHD) equilibrium and stability, and practical coil design. Experimental confirmation of this strategy has already achieved notable milestones. Optimized neoclassical transport has been confirmed in the Helically Symmetric eXperiment (HSX)¹ and in Wendelstein 7-X (W7-X).² The latter has also achieved high performance with respect to low turbulence-induced transport,³ island-divertor operation,⁴ and steady-state operation.⁵ Optimization of MHD equilibria and stability through design or through operational flexibility were realized earlier in the W7-AS and Large Helical Device (LHD) experiments⁶ and were central to the W7-X design.⁷ Other optimized stellarators are presently in the design phase.^{8,9}

Effective optimization relies on verified models of the underlying physical properties. For some of the targeted properties, first-principles are incorporated directly, such as the Biot-Savart law for computing magnetic field when designing coils.¹⁰ For other properties, such as MHD stability, proxies, limiting behavior,¹¹ and idealized criteria¹² are more practical when evaluating many configurations. When the most favorable designs are identified, more comprehensive simulation

can be applied to confirm expectation and to indicate where further improvements are possible. In particular, non-ideal, time-dependent MHD simulation, which is the focus of the contributions presented here, can be used to assess resistive and nonlinear effects for these designs and to help interpret experimental results.

Verification is an important step to establish the accuracy and reliability of specific implementations of models. Numerical results can be compared with analytical solutions for relatively simple tests, or benchmarked against results from previously verified codes for more representative applications. Benchmarking is also useful for predicting deviations between different numerical models used for the same application. One example is the benchmarking of asymmetric vertical displacement events (AVDEs) in tokamak simulation with the three extended-MHD codes JOEKE, M3D-C1, and NIMROD.¹³ While the reduced-MHD model of JOEKE describes all significant effects produced by the full-MHD computations in this application, its modeling does not allow toroidal current to transfer between the plasma and the wall over the toroidal angle. It is important to account for such model differences when simulating experimental conditions or evaluating a new design. By including a broad range of configurations and physical parameters, benchmarking can also be used to identify the regimes where different approximations and methods are most accurate and effective. This can guide further appli-

cation of the codes in the future.

In this paper, we benchmark the recently completed NIMSTELL code¹⁴ with the stellarator version of JOEK^{15,16} and the CASTOR3D code^{17,18} for a variety of stellarator configurations and MHD instabilities. NIMSTELL is a stellarator redevelopment of the extended-MHD code NIMROD.¹⁹ Similar to NIMROD, NIMSTELL uses a semi-implicit, leapfrog time advance and spectral elements in the poloidal plane. Extensions beyond NIMROD's capabilities include the Fourier-series representation of toroidally asymmetric geometries and steady (equilibrium) fields, and the option to use vector potential instead of the magnetic field as a time-dependent field. A detailed discussion of NIMSTELL's numerical methods, memory optimization, and initial verification tests is presented in Ref. 14. For the aims of this article, we just note that the combination of spectral elements and Fourier toroidal representation allows spectral convergence in all spatial dimensions, which is unique among stellarator MHD codes. The primary objective of the computations reported here is to benchmark the NIMSTELL implementation in representative conditions by comparing results on conventional and quasi-symmetric stellarator configurations with results from JOEK and CASTOR3D. Although NIMSTELL and JOEK have full nonlinear capability, our benchmarking focuses on linear computations, where distinct MHD responses are most readily apparent and tested.

We note that NIMSTELL and JOEK evolve their respective MHD models in time, whereas CASTOR3D solves for the normal modes of the single-fluid MHD equations. Thus, the use of CASTOR3D in this benchmark allows us to compare the linear growth rates of MHD instabilities obtained by time-dependent codes with direct calculations of eigenvalues. For linear computations, eigenvalue computation can be more informative when used to determine more than one eigenmode, whereas the time-asymptotic linear behavior of time-dependent computation only finds the fastest-growing eigenmode.

The time-dependent codes used in this work are relatively recent implementations for stellarators. Starting more than two decades ago, reduced-MHD computations with the NORM code were applied to study nonlinear interchange and profile relaxation in LHD.^{20,21} In addition, two-fluid and hot-particle effects in stellarators were modeled with the finite-element version of M3D during that period.²² Since then, the MIPS code has been used to study nonlinear ballooning,²³ the effects of resonant magnetic perturbation in heliotron configurations,²⁴ and MHD activity excited by pellet injection.²⁵ Core density collapse events in W7-X and the nonlinear evolution of high- n and low- n MHD modes have also been studied with MIPS.²⁶ The code was recently extended to include parallel thermal conduction and used to model a core density collapse event in LHD.²⁷ The relatively recent stellarator-geometry development of M3D-C1 was first applied to nonlinear relaxation of a W7-X VMEC equilibrium with near-zero pressure and on the loss of flux surfaces in a high beta W7-AS VMEC equilibrium.²⁸ In the second case, qualitative agreement was found with results²⁹ from the PIES code. Current-driven sawtooth-like crashes in W7-X were

modeled in a physics study, and semi-quantitative agreement with experiments was found.³⁰ M3D-C1 has also been used to examine nonlinear saturation of ballooning modes in the standard EIM configuration of W7-X, as well as a low- n interchange mode in an alternative TEH configuration.³¹ A parametric study with respect to the intensity of an external heating profile examined the relation between beta limits of the LHD standard configuration and the behavior of low- n MHD modes.³² A quasi-axisymmetric configuration was also modeled, where pressure-driven instabilities led to the development of chaotic magnetic field.³³

The JOEK code that is used in this work was previously extended to support reduced-MHD stellarator calculations.¹⁶ It was benchmarked through comparison with the eigenvalue code CASTOR3D¹⁷ for the (2, 1) tearing mode and finite- n ballooning in a W7-A configuration, and these are two of the applications considered in this article. JOEK's nonlinear evolution on the tearing case demonstrated saturation of magnetic island growth. JOEK was further extended to include parallel velocity and benchmarked against CASTOR3D for the (2, 1) tearing mode in W7-AS and interchange modes in a modified W7-A configuration. The development and benchmarking, and the code's application to the soft-beta limit in W7-AS, are reported in Ref. 34. JOEK has also been used to model low- n interchange modes in W7-X-like and W7-AS configurations, where a comparison was made with quasi-interchange modes in tokamaks.³⁵

Evidently, stellarator-MHD codes may be used to model a variety of stellarator configurations and phenomena. Thus, in this work, each benchmark case is selected for testing specific aspects of the codes that hold significance in physically relevant computations. The purpose of each case, along with the organization of this paper, is described below.

Section 2 outlines the single fluid MHD model in NIMSTELL and briefly summarizes the models in CASTOR3D and JOEK. Section 3 describes four different benchmark setups. First, the (2, 1) tearing mode in a W7-A configuration serves as a test for resistive dynamics. This is followed by a resistive ballooning mode in W7-A to test localized pressure-driven dynamics that are relevant for stellarator experiments and design studies. The rotating-ellipse stellarator geometry of these cases is essentially represented by just two toroidal Fourier components, one symmetric and one asymmetric. Contemporary optimized configurations with representative strong shaping are considered next. The third case consists of the (2, 1) tearing mode in a quasi-axisymmetric configuration. Lastly, a global interchange mode is modeled as a test for pressure-driven dynamics in a quasi-helically symmetric configuration. The nonlinear capability of NIMSTELL is demonstrated by calculating the evolution of the global interchange mode. Finally, section 4 includes a discussion of the results, limitations of the benchmarking computations, and conclusions.

II. MHD MODEL

NIMSTELL presently solves a single-fluid MHD model comprised of three fluid equations, Faraday's law, and the resistive-MHD Ohm's law. The fluid equations are given by:

$$\frac{\partial n}{\partial t} + \nabla \cdot n \mathbf{V} = \nabla \cdot (D \nabla n), \quad (1)$$

$$mn \left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} \right) = -\nabla p + \frac{1}{\mu_0} (\nabla \times \mathbf{B}) \times \mathbf{B} - \nabla \cdot \bar{\Pi}, \quad (2)$$

$$\frac{n}{\gamma - 1} \left(\frac{\partial T}{\partial t} + \mathbf{V} \cdot \nabla T \right) = -nT \nabla \cdot \mathbf{V} + \nabla \cdot (\bar{\kappa} \cdot \nabla T), \quad (3)$$

where n is the electron particle density, m is the effective mass per electron ($m \approx m_i$ for singly charged ions of mass m_i), $\mathbf{V}$ is the mass-averaged flow velocity, p is the total thermal pressure, $\mathbf{B}$ is the magnetic field, and $T = T_e = T_i$ is the plasma temperature. The diffusive terms include the particle diffusivity D , the viscous stress tensor $\bar{\Pi}$, and the thermal conductivity tensor $\bar{\kappa}$. The constants γ and μ_0 represent the ratio of specific heats and the vacuum permeability, respectively.

Equations (1)-(3) are solved for the time-dependent fields $n, \mathbf{V}$ and T , where $\mathbf{V}$ is represented by its cylindrical components. Although different forms of the stress tensor $\bar{\Pi}$ are available in NIMSTELL, we use the following form for the present calculations:

$$\bar{\Pi} = -m_i n_0 \left(\frac{1}{2} v_{\parallel} \hat{\mathbf{b}} \cdot \bar{\mathbf{W}} \cdot \hat{\mathbf{b}} (3\hat{\mathbf{b}}\hat{\mathbf{b}} - \bar{\mathbf{I}}) + v_{iso} \bar{\mathbf{W}} \right),$$

$$\bar{\mathbf{W}} = \nabla \mathbf{V} + (\nabla \mathbf{V})^T - \frac{2}{3} \bar{\mathbf{I}} (\nabla \cdot \mathbf{V}),$$

where n_0 is a constant density parameter, and v_{iso} and $v_{\parallel}$ are the isotropic and parallel viscous diffusivities, respectively. The thermal conductivity tensor consists of the parallel and perpendicular components relative to the magnetic field, $\bar{\kappa} = \kappa_{\parallel} \hat{\mathbf{b}}\hat{\mathbf{b}} + \kappa_{\perp} (\bar{\mathbf{I}} - \hat{\mathbf{b}}\hat{\mathbf{b}})$. At present, the thermal conductivities are independent of T and linearly proportional to n . Temperature-dependent closure relations will be implemented in future development.

The magnetic representation in NIMSTELL may be in terms of components of the magnetic field $\mathbf{B}$ or vector potential $\mathbf{A}$, and both options are applied in some of the computations of Section IV. The former option evolves the cylindrical components of $\mathbf{B}$ using the combined Faraday/Ohm's equations:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}) - \nabla \times \left(\frac{\eta}{\mu_0} \nabla \times \mathbf{B} \right) + \kappa_{divb} \nabla (\nabla \cdot \mathbf{B}), \quad (4)$$

where η is the electrical resistivity and κ_{divb} is a numerical diffusivity for the divergence of the magnetic field. Divergence cleaning is necessary for magnetic fields represented by H^1 spectral elements, and this diffusive method has proven to be effective in NIMROD computations when using elements

of polynomial degree two and larger.¹⁹ Alternatively, the following form of the electric-field relation $\mathbf{E} = -\partial \mathbf{A} / \partial t - \nabla \chi$ combined with Ohm's law can be used to evolve the vector potential:

$$\bar{\sigma} \cdot \frac{\partial \mathbf{A}}{\partial t} = \bar{\sigma} \cdot \mathbf{V} \times (\nabla \times \mathbf{A}) - \nabla \times (\nabla \times \mathbf{A}) - \bar{\sigma} \cdot \nabla \chi, \quad (5)$$

$$\nabla^2 \chi = d_A \nabla \cdot \mathbf{A},$$

where $\nabla \times \mathbf{A} = \mathbf{B}$, $\bar{\sigma}$ is the electrical conductivity tensor, χ is a scalar potential that sets the gauge, and d_A is effectively a damping coefficient for $\nabla \cdot \mathbf{A}$. The NIMSTELL implementation uses H^{curl} spectral elements to ensure $\nabla \cdot \mathbf{B} = 0$ within elements, continuity of the normal component of $\mathbf{B}$ across element interfaces, and approximate adherence to the Coulomb gauge.¹⁴

For the benchmarking computations included in this work, the NIMSTELL equations are linearized about a steady-state solution, which is taken to be a static equilibrium that satisfies $(\nabla \times \mathbf{B}) \times \mathbf{B} = \mu_0 \nabla p$. Only the perturbations are then advanced in time, starting from small-amplitude seed fields. Over time, the fastest growing eigenmode dominates the solution.

CASTOR3D directly solves the eigenvalue problem given by the following equations:^{17,18}

$$\lambda \rho = -\rho_0 \nabla \cdot \mathbf{V} - \mathbf{V} \cdot \nabla \rho_0, \quad (6)$$

$$\lambda \rho_0 \mathbf{V} = -\nabla (\rho_0 T + \rho T_0) / m + \frac{1}{\mu_0} (\nabla \times \mathbf{B}_0) \times \mathbf{B} + \frac{1}{\mu_0} (\nabla \times \mathbf{B}) \times \mathbf{B}_0, \quad (7)$$

$$\lambda T = -\mathbf{V} \cdot \nabla T_0 - (\gamma - 1) T_0 \nabla \cdot \mathbf{V}, \quad (8)$$

$$\lambda \mathbf{A} = \mathbf{V} \times \mathbf{B}_0 - \frac{\eta}{\mu_0} \nabla \times (\nabla \times \mathbf{A}), \quad (9)$$

$$\mathbf{B} = \nabla \times \mathbf{A}, \quad (10)$$

where $\rho = mn$ is the mass density, the subscript 0 denotes equilibrium fields, and the equilibrium-flow terms in CASTOR3D are not shown. The time dependence is assumed to be $\sim \exp(\lambda t)$, where λ represents a complex eigenvalue. The gauge is fixed by setting the scalar potential to zero.

In contrast to the full-MHD models in NIMSTELL and CASTOR3D, JOREK computations considered here use a set of reduced-MHD equations.^{16,36} The time-dependent fields include the mass density, a velocity stream function, the plasma temperature and a magnetic flux function. The initial conditions are derived from an equilibrium and no separation of the steady-state is applied. The velocity and the magnetic field are modeled in terms of a stream function and a magnetic flux function whose gradients are crossed with the vacuum magnetic field. This removes fast and slow compressive waves from the system. In addition to particle diffusivity, viscosity, resistivity and thermal conductivity, the model includes hyperviscosity and hyperresistivity. For completeness, the equations are provided in Appendix A.

III. NUMERICAL METHODS

In NIMSTELL, the fields $n, \mathbf{V}, T$ and $\mathbf{B}$ are split into their steady-state and time-dependent components, which allows strictly linear computation. Nonlinear terms are implemented without an ordering assumption. Equations for the time-dependent components are integrated using a semi-implicit, leapfrog method.¹⁹ All steady-state and time-dependent fields are spatially represented using a combination of spectral elements in the poloidal plane and finite Fourier series over a generalized toroidal angle, ζ . Polynomials of the same degree are used as basis functions in the 2-D spectral elements for all steady-state fields and for the time-dependent density and temperature. The degree of the polynomials for the time-dependent velocity and magnetic field can be chosen separately, offering greater numerical flexibility. The use of this capability will be noted in the benchmark cases where relevant.

Element coordinates and the generalized toroidal angle are mapped to cylindrical coordinates (R, Z, ϕ) using the same 2-D spectral element bases as the equilibrium fields and Fourier series over ζ . Computing the spatial derivatives of fields from the derivatives with respect to element coordinates requires multiplication by the inverse Jacobian of the coordinate mapping. Since the mapping from elements to physical coordinates is represented by finite Fourier series, in principle, its inverse contains an infinite number of Fourier harmonics. Thus, to avoid aliasing errors in strongly shaped configurations, a generalized integration rule is used in NIMSTELL. Fourier transforms are calculated on a toroidal grid of 2^l poloidal planes, where the Fourier series is truncated after the harmonic number $n = \lfloor (2^l - 1)/(n_d + 1) \rfloor$. Here, n_d is the dealiasing parameter, such that $n_d = 2$ corresponds to the usual 3/2 rule for quadratic nonlinearities, and $n_d \geq 3$ results in stronger dealiasing.

The mesh mapping and steady-state fields are calculated by an interface between NIMSTELL and the ideal MHD equilibrium code DESC.³⁷ Except near the magnetic axis, the spectral elements conform to the nested flux surfaces of the DESC equilibrium on any poloidal plane. To avoid having quadrilateral elements that degenerate into triangles, the meshing near the magnetic axis transitions to a rectangular arrangement, as shown in Fig. 2 of Ref. 14.

The time-dependent fields in NIMSTELL are initialized with a small perturbation amplitude, where the spatial variation is determined by user-defined parameters. The perturbation projects onto many eigenmodes of the system, and the fastest-growing instability eventually dominates the time-dependent fields.

In the NIMSTELL computations presented here, the no-slip boundary condition is used for velocity. The magnetic field is constrained to be tangential to the last closed flux surface (LCFS), *i.e.*, a perfectly conducting wall is assumed. Dirichlet and zero-flux boundary conditions are used for the temperature and density, respectively. The viscosity and electrical resistivity are constant, uniform, and isotropic. Particle diffusivity and thermal conductivity are set to zero.

The numerical implementations in CASTOR3D and

JOREK differ from that in NIMSTELL. Here, we outline the key differences that may be relevant to the benchmark results. CASTOR3D represents all spatially varying fields through 2D Fourier series in Boozer coordinates, and a combination of quadratic and cubic Hermite-polynomial elements in the flux-normal direction.¹⁷ Unlike NIMSTELL, the number of harmonics in the Fourier expansions for the equilibrium fields are specified separately from those for the solution fields. While the equilibrium fields use all poloidal and toroidal harmonics up to some truncation limits, the solution fields can flexibly use a given set of helical harmonics $\{(m, n)\}$. Typically, Fourier series expansions in Boozer angles require truncation at m and n values that are several times their maximum values in the input VMEC or GVEC equilibrium. In CASTOR3D, the component of velocity normal to the LCFS is set to zero. Boundary conditions on the magnetic field set the tangential components of $\mathbf{A}$ to zero at the plasma surface to represent a close-fitting conducting wall. The temperature and density perturbations are set to zero at the boundary.

In JOREK, spatial discretization is based on G^1 -continuous quadrilateral finite elements in the poloidal plane and finite Fourier series in the toroidal direction. Since the equilibrium is represented by the initial values of the time-dependent fields, the toroidal resolutions of the two are not independent. Nonetheless, the mesh geometry retains the toroidal resolution of the equilibrium solution. For the time advance, the four reduced-MHD scalar fields are solved implicitly using either the implicit Euler or the Crank-Nicolson scheme. This leads to smaller temporal truncation error than the semi-implicit scheme in NIMSTELL for a given time-step size. The associated computational cost in JOREK is due to solving larger algebraic systems at each step. The linear growth rates and eigenmodes are inferred from the linear phase of nonlinear computations. Dirichlet boundary conditions are used for the mass density ρ and the pressure p . The boundary condition for the magnetic flux function Ψ (see Appendix A) in stellarator geometries requires solving a surface differential equation. The method used in JOREK computations is described in Section IV of Ref. 16. The velocity stream function Φ is set to zero at the boundary, ensuring that the velocity normal to the boundary is zero. This differs from the no-slip boundary condition in NIMSTELL.

IV. BENCHMARK CASES

Sections IV A-IV D describe the equilibria, physical and numerical parameters, and the results for four benchmarks. The W7-A tearing and ballooning mode benchmarks in Sections IV A and IV B are based on previous benchmarks between JOREK and CASTOR3D.¹⁶ We note that the CASTOR3D calculations presented here use greater numerical resolution, and the eigenvalues are more accurate, than those reported in Ref. 16. Sections IV C and IV D describe benchmarks for the (2, 1) tearing mode in a QA configuration and a global interchange mode in a QH configuration, respectively. In Section IV A, NIMSTELL computations are run with both of its magnetic representations, namely, components of the

magnetic field $\mathbf{B}$ in H^1 space and the vector potential $\mathbf{A}$ in H^{curl} space. All other sections report results with only the magnetic-component representation.

A. Tearing mode in W7-A

The W7-A configurations presented here have an LCFS with a rotating-ellipse cross section. The major and minor axes are 0.1178 m and 0.09131 m, respectively. The major radius is 1.99 m, giving an aspect ratio of 20. The configuration has five field periods. All W7-A computations use a pressure profile of the form:

$$p(s) = p_a - (p_a - p_b)s, \quad (11)$$

where s is the normalized toroidal flux, p_a is the pressure at $s = 0$, and p_b is the pressure at $s = 1$. For the tearing mode benchmark, three configurations are generated with $p_a = 1$, 10, and 100 Pa and $p_b/p_a = 10^{-2}$. The resulting volume averaged β (%) values are 2.3×10^{-5} , 2.3×10^{-4} and 2.3×10^{-3} , respectively. The following current profile is used for all W7-A configurations:

$$I_{\text{tor}}(s) = 17.5 (3s - 3s^2 + s^3) \text{ kA}. \quad (12)$$

The resulting rotational transform $t(s)$ takes the rational value of $1/2$ at $s = 0.59$. Due to the very small β values, the rotational transform varies negligibly between configurations with different β s. A uniform particle density of $n = 2 \times 10^{20} \text{ m}^{-3}$ is assumed for both electrons and ions, and the equilibrium temperature is set as $T_e = T_i = p/2n$. The ion particle mass is set to the mass of a proton. With $B \approx 2.4 \text{ T}$, the Alfvén time is $\tau_A \approx 0.54 \mu\text{s}$.

The NIMSTELL poloidal mesh consists of 30 radial and 32 poloidal biquartic elements in an annular region surrounding a topologically rectangular region of 64 elements. The toroidal Fourier transforms use 128 poloidal planes and retain mode numbers up to $n = 31$. The time step size is $6.484 \times 10^{-7} \text{ s}$. Spatial and temporal resolution parameters are determined through convergence such that increasing the resolution does not change the growth rate by more than 0.5%. The H^1 representation of magnetic-field components and the H^{curl} representation of vector potential are applied in separate NIMSTELL series of computations for this case. JOREK computations use 41 and 48 nodes in the radial and poloidal directions, respectively. The toroidal grid consists of 40 poloidal planes and toroidal mode numbers up to $n = 10$. The time step size is $1.621 \times 10^{-5} \text{ s}$. In CASTOR3D, 101 radial grid points spaced uniformly in $\sqrt{s}$ are used for the Hermite-element basis. Equilibrium fields are represented with poloidal and toroidal harmonics up to $m = 25$ and $n = 15$, respectively. The solution fields are represented by Fourier series containing the harmonics $-3 \leq m \leq 7$ and $n = 1, 4, 6, 9, 11$.

While no viscosity or hyperviscosity is used in CASTOR3D, viscosity in NIMSTELL and hyperviscosity in JOREK are nonzero. For NIMSTELL calculations, the value of viscosity is $\mu = 5.16 \times 10^{-11} \text{ kg/m-s}$. This is the minimum value that avoids numerical instability with NIMSTELL's

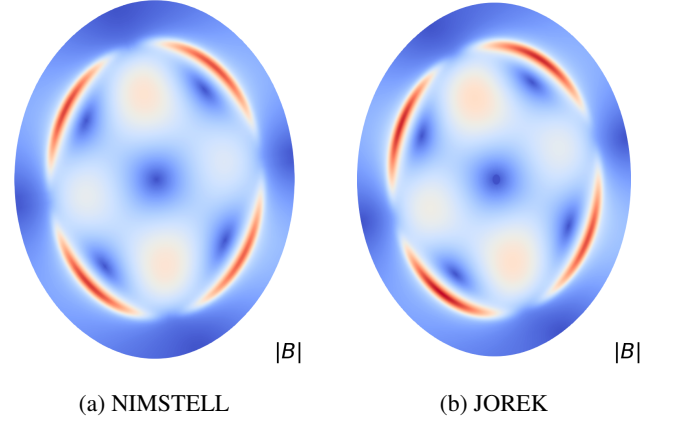


FIG. 1: The $(2, 1)$ tearing mode eigenfunction at $\beta = 2.3 \times 10^{-5}\%$ and $\eta = 1.938 \times 10^{-6} \Omega\text{m}$ is compared in terms of the magnitude of the perturbed magnetic field.

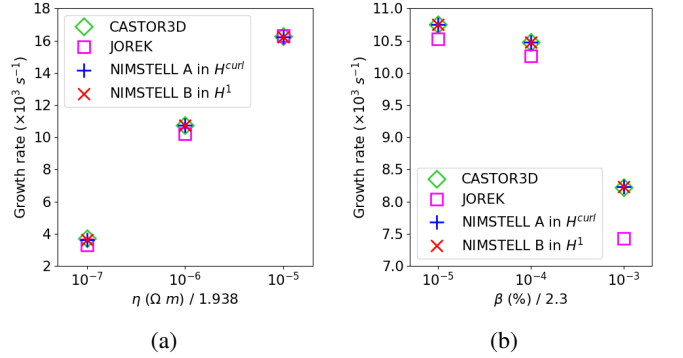


FIG. 2: Growth rates of the $(2, 1)$ tearing mode in a W7-A configuration are computed using NIMSTELL, JOREK and CASTOR3D at (a) $\beta = 2.3 \times 10^{-5}\%$ for three values of resistivity η , and (b) $\eta = 1.938 \times 10^{-6} \Omega\text{m}$ for three values of β .

vector-potential representation on the spatial grid used for resolving the tearing mode in the $\eta = 1.938 \times 10^{-7} \Omega\text{m}$, $\beta = 2.3 \times 10^{-5}\%$ case, which has the thinnest tearing layer among all cases. The requirement of a grid-dependent minimum level of resistive diffusion with this representation is discussed in Ref. 14. The hyperviscosity in JOREK is set to $\mu_h = 2.90 \times 10^{-15} \text{ kg-m/s}$ for the $\beta = 2.3 \times 10^{-5}\%$ cases, and $\mu_h = 7.25 \times 10^{-15} \text{ kg-m/s}$ for the $\beta = 2.3 \times 10^{-4}\%$ and $\beta = 2.3 \times 10^{-3}\%$ cases. Additionally, a hyperresistivity of $\eta_h = 9.69 \times 10^{-14} \Omega\text{m}^2$ is used in the $\eta = 1.938 \times 10^{-7} \Omega\text{m}$, $\beta = 2.3 \times 10^{-5}\%$ case. More details about the selection of parameters for JOREK can be found in Ref. 16.

Figure 1 shows the tearing-mode eigenfunction obtained in NIMSTELL and JOREK. The $(2, 1)$ structure is apparent, and the scale of the tearing layer is the same for the two codes. Figure 2 compares the growth rates of the $(2, 1)$ tearing mode calculated using CASTOR3D, JOREK and NIMSTELL for two parameter scans. The largest difference between CASTOR3D and NIMSTELL growth rates is 1.5% and occurs at

$\eta = 1.938 \times 10^{-7} \Omega\text{m}$, $\beta = 2.3 \times 10^{-5}\%$. For the other parameters, the results of the two codes agree to within 0.3%. We attribute the more pronounced discrepancy in the lowest resistivity case to the viscosity in NIMSTELL computations. If the viscosity is reduced from $\mu = 5.16 \times 10^{-11} \text{ kg/m-s}$ to $\mu = 2.58 \times 10^{-11} \text{ kg/m-s}$, NIMSTELL shows slightly better agreement with CASTOR3D, but requires greater spatial resolution to avoid numerical instability with the vector-potential representation. CASTOR3D and JOEY growth rates differ by up to 11%, with the largest difference at $\eta = 1.938 \times 10^{-7} \Omega\text{m}$, $\beta = 2.3 \times 10^{-5}\%$. This discrepancy is attributed to the use of reduced-MHD in the JOEY computations, in contrast to the full-MHD modeling in NIMSTELL and CASTOR3D. In addition to the cross-code comparisons, Figure 2 shows consistency in the results from the two representations for magnetic-field information in NIMSTELL.

B. Ballooning mode in W7-A

Configurations used for the ballooning mode benchmark have the same LCFS shape that is described in section IV C. The pressure profile is given by Eqn. 11 and the current profile by Eqn. 12. Equilibria are generated at $\beta = 0.11\%$ and $\beta = 0.21\%$ by setting (p_a, p_b) to (5 kPa, 1 Pa) and (10 kPa, 100 Pa), respectively. The particle density and temperature are set as per section IV A.

The ballooning mode with $n = 10$ as its dominant harmonic is considered, which is referred to as the $n_{dom} = 10$ mode. Since the configuration has five field periods, all three codes take advantage of the periodicity by modeling one-fifth of the torus. As a consequence of toroidal coupling due to asymmetry, the toroidal spectrum of this eigenmode also includes $n = 0, 5, 15, 20, \dots$ in addition to the dominant harmonic. However, in the absence of numerical damping, when $n > 10$ harmonics are included in time-dependent MHD calculations, ballooning modes with $n_{dom} > 10$ grow faster than the $n_{dom} = 10$ mode and obscure the information of interest about the latter. On the other hand, to resolve the eigenfunction of the $n_{dom} = 10$ mode accurately, its $n > 10$ harmonics must be included.

Computations with the three codes tackle this difficulty in two different ways. Fourier series are truncated at $n = 10$ for the time-dependent fields in JOEY and for the eigenfunctions in CASTOR3D, which avoids the $n_{dom} > 10$ ballooning modes. This is based on an estimation that inclusion of the $n = 15$ harmonic changes the growth rate insignificantly.¹⁶ In NIMSTELL, more harmonics of the time-dependent fields are included, but the $n > 10$ harmonics of density and temperature are artificially damped with a damping coefficient of $7.7 \times 10^4 \text{ s}^{-1}$, which slows the $n_{dom} > 10$ ballooning modes while retaining more of the toroidal spectrum of the $n_{dom} = 10$ mode. If a smaller damping coefficient is used, the $n_{dom} = 15$ ballooning mode grows faster than the desired $n_{dom} = 10$ mode. In the $\beta = 0.21\%$, $\eta = 5.814 \times 10^{-7} \Omega\text{m}$ case, increasing the damping coefficient by 20% changes the growth rate of the $n_{dom} = 10$ ballooning mode by only 1.1%. This indicates that the artificial damping has a small effect on the

ballooning mode in question.

Because the mode is localized near the LCFS, these NIMSTELL calculations model an annular region between $s = 0.04$ and $s = 1$ without covering the neighborhood of the magnetic axis. The viscosity is fixed at $\mu = 5.16 \times 10^{-11} \text{ kg/m-s}$, the time step is $1.621 \times 10^{-7} \text{ s}$, and only the H^1 representation for magnetic-field components is considered. The polynomials for all fields except velocity are biquartic. The flow-velocity uses polynomials of one degree lower, *i.e.*, bicubic, to achieve convergence on pressure-driven dynamics from the stable side (see Section 4.5 of Ref. 38) without numerical stabilization terms. Poloidal and toroidal resolution parameters are determined through convergence tests conducted for the case with $\beta = 0.21\%$ and $\eta = 1.938 \times 10^{-7} \Omega\text{m}$.

Figure 3 (top panel) shows convergence with respect to the number of elements in the poloidal direction, where 48 radial elements and toroidal harmonics up to $n = 25$ based on 32 poloidal planes in one field period are used. The mesh is packed near the boundary where the ballooning mode is localized. While either 256 or 384 elements in the poloidal direction may suffice, the latter is chosen for the remaining cases. The ballooning mode is found to be resolved in all three directions, since the growth rate differs by only 0.25% compared to a calculation with 64 and 512 elements in the radial and poloidal directions, respectively, and toroidal harmonics up to $n = 30$.

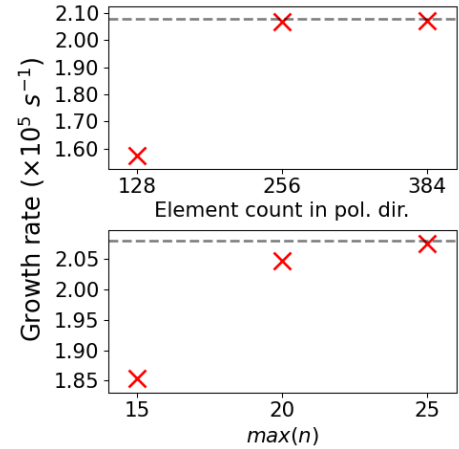


FIG. 3: Numerical convergence of the $\beta = 0.21\%$, $\eta = 1.938 \times 10^{-7} \Omega\text{m}$ ballooning-mode growth rate from NIMSTELL is shown with respect to the number of finite elements in the poloidal direction (top), and the toroidal resolution (bottom), where $\max(n)$ is the highest harmonic retained in the calculation. Dashed lines show the growth rate calculated with the highest attempted resolution of 64 and 512 elements in the radial and poloidal directions, respectively, and toroidal harmonics up to $n = 30$.

JOEY computations use 61 and 72 nodes in the radial and poloidal directions, respectively, and toroidal mode numbers $n = 0, 5, 10$ are retained in the time-dependent fields. The time step size is $6.484 \times 10^{-7} \text{ s}$. The hyperviscosity is fixed at $\mu_h = 7.25 \times 10^{-15} \text{ kg/m-s}$, while the viscosity is set to zero.

The equilibrium resolution in CASTOR3D is the same as that in W7-A tearing cases. The solution fields are represented by 101 radial grid points, and Fourier series containing the harmonics $9 \leq m \leq 16$ for $n = 5$, $18 \leq m \leq 32$ for $n = 10$, and $35 \leq m \leq 44$ for $n = 15$. These pairs of harmonics represent the most significant Fourier components of the ballooning eigenfunctions, which have a helical structure determined by the rotational transform profile.

The number of elements over the poloidal direction used in NIMSTELL computations is considerably larger than that used in the JOREK computations. Together with the $n > 10$ toroidal resolution, this allows the NIMSTELL computation to produce smaller-scale features of the eigenmode than the JOREK result, as shown in Figure 4. This is further illustrated in Figure 5, where the NIMSTELL eigenfunction has been separated into its $n \leq 10$ and $n > 10$ harmonics. The small-scale features are more prominent in the higher harmonics, and do not align with the mesh. We surmise that they are part of the true eigenfunction. Figure 3 (bottom panel) shows how limiting the toroidal representation for the equilibrium, geometry, and time-dependent fields affects the computed NIMSTELL growth rate.

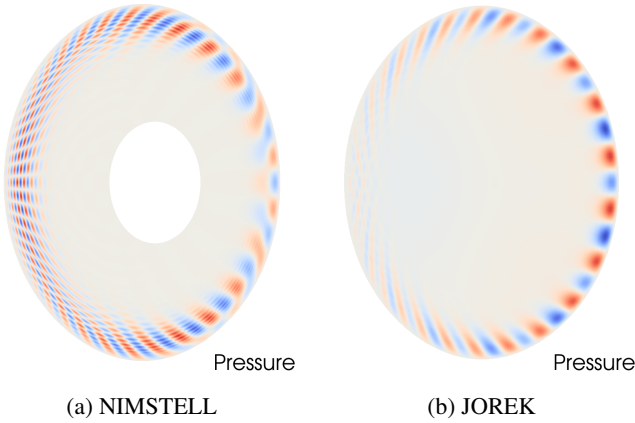


FIG. 4: The ballooning-mode eigenfunction at $\beta = 0.21\%$ and $\eta = 1.938 \times 10^{-7} \Omega\text{m}$ is compared in terms of the perturbed pressure. Values are normalized to the range $[-1, 1]$.

Figure 6 shows a comparison between the growth rates calculated by the three codes in four different cases. The largest difference in growth rates between NIMSTELL and CASTOR3D is 5.8% and that between JOREK and CASTOR3D is 6.2%, both of which occur at $\beta = 0.11\%$ and $\eta = 1.938 \times 10^{-7} \Omega\text{m}$. The CASTOR3D growth rate is smaller than the NIMSTELL result and larger than the JOREK result. While reduced- vs. full-MHD influences the comparison with JOREK results, the different approaches to limiting the ballooning spectrum likely affects the comparison of NIMSTELL results.

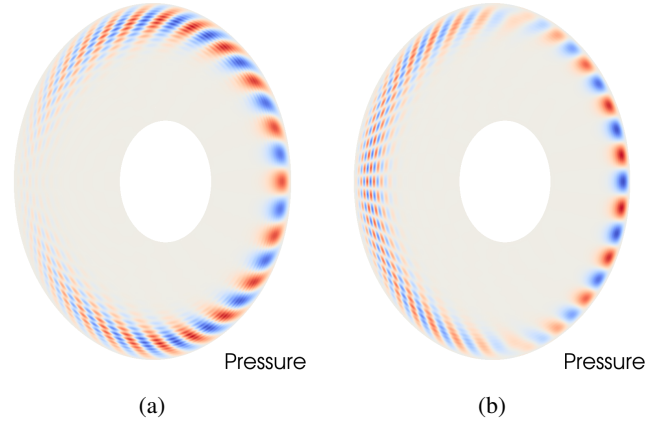


FIG. 5: The perturbed pressure in Figure 4a is separated into two sets of its harmonics, (a) $n = 0, 5, 10$, and (b) $n = 15, 20, 25$.

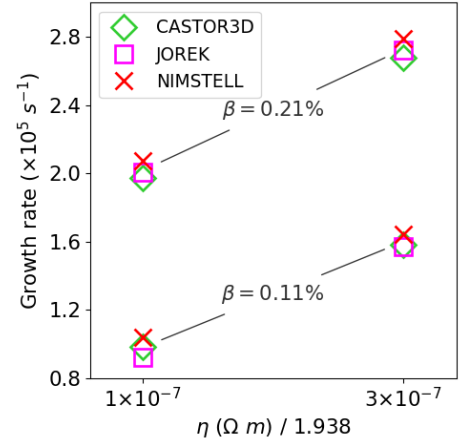


FIG. 6: Growth rates of the ballooning mode with $n = 10$ as the dominant toroidal harmonic are computed using NIMSTELL, JOREK and CASTOR3D for two values of resistivity η at two different β 's.

C. Tearing mode in QA

To create a tearing-mode case in a strongly shaped geometry, an existing vacuum QA configuration³⁹ having two field periods and an aspect ratio of 6.0 is modified by adding an artificial current profile given by

$$I_{tor}(s) = 19.0s - 12.8s^2 + 3.84s^3 \text{ kA}.$$

As shown in Figure 7, the artificial current modifies the original rotational transform profile such that a rational surface with $t(s) = 1/2$ is admitted at $s = 0.29$. A uniform particle density of $n = 2 \times 10^{20} \text{ m}^{-3}$ and an ion mass of $m_i = m_p = 1.67 \times 10^{-27} \text{ kg}$ sets the mass density, as in the W7-A computations.

The $(2, 1)$ tearing mode is calculated for a range of resistivities with NIMSTELL, CASTOR3D and JOREK. For cases

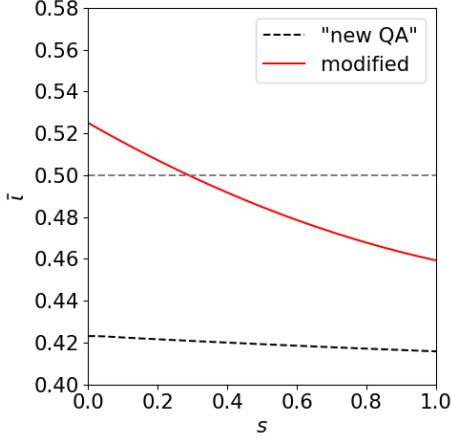


FIG. 7: Rotational transform profiles of the Landreman-Paul "new QA" configuration and a modified equilibrium with an artificial toroidal current are compared. Unlike the reference, the modified profile admits an $t = 0.5$ surface.

with $\eta \geq 1.938 \times 10^{-8} \Omega\text{m}$, the NIMSTELL mesh consists of 31 radial and 16 poloidal biquartic elements for all fields in an annular region surrounding a 16-element mesh over the vicinity of the magnetic axis. The representation of magnetic field is via the H^1 expansion for its vector components. A more refined mesh with 62 radial and 32 poloidal biquartic elements surrounding a region of 64 elements over the axis is necessary for resolving the mode at $\eta \leq 1.938 \times 10^{-9} \Omega\text{m}$. Both meshes are packed in the annular region near the $t = 1/2$ surface. Toroidal mode numbers up to $n = 21$ are retained, which are calculated from a grid of 128 poloidal planes. The time step sizes used for the $\eta \geq 1.938 \times 10^{-8} \Omega\text{m}$ and $\eta \leq 1.938 \times 10^{-9} \Omega\text{m}$ cases are $8.1 \times 10^{-6} \text{ s}$ and $1.9 \times 10^{-7} \text{ s}$, respectively. The smaller time step is necessary to help the linear solver converge, since lower resistivities result in more ill-conditioned matrices. Convergence with respect to the spatial and temporal resolution parameters is tested by increasing the resolution until the growth rates change by less than 1.5%.

We denote the poloidal resolution in JOREK by the ordered pair (n_s, n_θ) , where n_s and n_θ are the numbers of nodes in the radial and poloidal directions, respectively. Poloidal resolutions for the resistivity scan are then given by (41, 48) for $\eta = 1.938 \times 10^{-6} \Omega\text{m}$, (41, 96) for $\eta = 1.938 \times 10^{-7} \Omega\text{m}$ and $1.938 \times 10^{-8} \Omega\text{m}$, and (281, 48) for $\eta = 1.938 \times 10^{-9} \Omega\text{m}$ and $1.938 \times 10^{-10} \Omega\text{m}$. The highest toroidal harmonic n and time-step size Δt are $n = 8$, $\Delta t = 1.2967 \times 10^{-4} \text{ s}$ for $\eta \geq 1.938 \times 10^{-8} \Omega\text{m}$, and $n = 10$, $\Delta t = 1.2967 \times 10^{-3} \text{ s}$ for $\eta \leq 1.938 \times 10^{-9} \Omega\text{m}$. In CASTOR3D, equilibrium fields are represented with 401 radial grid points, and poloidal and toroidal harmonics up to $m = 60$ and $n = 60$, respectively. The solution fields are represented by Fourier series containing the harmonics $-10 \leq m \leq 12$ and $n = 2k + 1$, $0 \leq k \leq 6$.

While the CASTOR3D computations do not use viscosity, a nonzero value is used in the NIMSTELL and JOREK computations. To keep the magnetic Prandtl number fixed, viscosity is varied proportionately with the resistivity. Informed by

the previous applications, we choose the lowest value of viscosity to be $\mu = 5.16 \times 10^{-14} \text{ kg/m-s}$ in the lowest resistivity case, $\eta = 1.938 \times 10^{-10} \Omega\text{m}$. This fixes the magnetic Prandtl number at $Pm = 10^{-3}$. For reference, the lowest resistivity of $\eta = 1.938 \times 10^{-10} \Omega\text{m}$ corresponds to a Lundquist number of $S = 2.8 \times 10^8$.

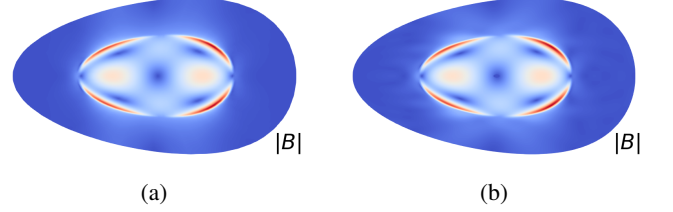


FIG. 8: The (2, 1) tearing mode eigenfunction in an equilibrium modified from the Landreman-Paul QA configuration is shown for the $\eta = 1.938 \times 10^{-8} \Omega\text{m}$ case in terms of the magnitude of the perturbed magnetic field in (a) NIMSTELL, and (b) JOREK.

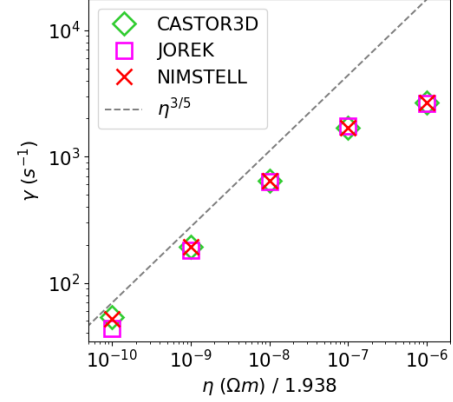


FIG. 9: Growth rates of the (2, 1) tearing mode in the modified Landreman-Paul QA configuration are computed using NIMSTELL, JOREK and CASTOR3D for a range of resistivity η .

Figure 8 compares contours of the magnetic field perturbation magnitude computed using NIMSTELL and JOREK for $\eta = 1.938 \times 10^{-8} \Omega\text{m}$. Agreement with respect to the mode shape and the tearing-layer width is apparent from the figure. Growth rates of the (2, 1) tearing mode calculated by all three codes are compared in Figure 9. As expected, the growth rates asymptote to the $\eta^{3/5}$ scaling as resistivity is decreased. The largest difference between the NIMSTELL and CASTOR3D growth rates is 3.0% and occurs at $\eta = 1.938 \times 10^{-10} \Omega\text{m}$. For other values of η , the difference is less than 1.0%. NIMSTELL and JOREK growth rates differ by 15% for $\eta = 1.938 \times 10^{-10} \Omega\text{m}$, and by less than 5.1% for other values of η .

At $\eta = 1.938 \times 10^{-10} \Omega\text{m}$, the tearing mode grows very slowly compared to the Alfvénic time scale ($\gamma\tau_A \approx 3 \times 10^{-5}$) and develops over a duration of $\sim 10^5 \tau_A$ in the time-dependent calculations, making it difficult to get a precise

value for the growth rate. The timescale separation also presents a challenge in the iterative eigenvalue calculations of CASTOR3D. These difficulties may contribute to the discrepancy in growth rates for the $\eta = 1.938 \times 10^{-10} \Omega m$ case.

An additional point on the QA tearing computations is that the residual force-balance error in the equilibrium is of sufficient magnitude to require special treatment in JOEREK, despite the residual force-density throughout the domain being within 10^{-4} of the volume-averaged magnetic pressure gradient. Because JOEREK is configured for nonlinear computations where the equilibrium is part of the initial condition for the time-dependent fields, any residual forces can contribute to the evolution of the plasma configuration on the Alfvénic timescale. This affects the tearing mode, which evolves on a slower timescale. Such an effect is absent in NIMSTELL computations for the linearized system of equations, which does not allow equilibrium fields to evolve. Noting that the equilibrium has two field periods, the even and odd toroidal Fourier harmonics are decoupled in the linear phase of JOEREK computations. The equilibrium fields reside in the even harmonics, and the eigenfunction of the (2, 1) tearing mode is represented with only the odd harmonics. To reproduce strictly linear behavior for the imperfect numerical equilibrium, JOEREK is modified to hold the even toroidal harmonics constant in these computations, which preserves the initial equilibrium fields while allowing the instability to evolve.

D. Interchange in QH

The aim of this section is to benchmark NIMSTELL for a pressure-driven instability in a configuration optimized for quasisymmetry. The configurations used here are modified from the $\beta = 2.5\%$, aspect ratio 6.5 QH configuration with self-consistent bootstrap current of Ref. 40. They have non-monotonic rotational transform profiles with their minimum values close to unity. Consequently, there exists a region of near-zero global magnetic shear where the rotational transform is close to a low-order rational number. This magnetic configuration admits global, nearly-resonant ideal interchange, or *infernal*,⁴¹ modes with $m/n = 1$, where m and n represent the dominant poloidal and toroidal mode numbers, respectively. The growth rate of such interchange modes has a much weaker dependence on the wavelength than the ballooning-mode growth rate in section IV B. Thus, unlike the ballooning-mode benchmark, numerical convergence can be obtained without relying on artificial damping.

Benchmark configurations are developed from the original QH equilibrium by prescribing the rotational transform profile

$$t(s) = t_0 + 0.103 (3.25 - s)(s - 0.49)^2,$$

where t_0 is the minimum value of this polynomial occurring at $s = 0.49$. Imposing this rotational transform affects the quasi-symmetry and violates self-consistency of the bootstrap current. However, the modified rotational transform profile is chosen to resemble the original in that it is non-monotonic and its minimum value is close to unity. These features are essential for the global interchange mode to occur. Compared

to the original equilibrium, the location of the minimum t is shifted away from the LCFS to avoid sharp gradients near the boundary, which facilitates numerical resolution. Moreover, the polynomial specification allows direct control over t_0 , a parameter that is varied in the present calculations.

The density and temperature in the original configuration vanish at the LCFS, which can lead to negative values in time-dependent calculations. For this reason, these profiles are slightly modified to have nonzero values on the LCFS while maintaining the same values on the axis:

$$n(s) = (2.2 - 2.0 s^5) \times 10^{20} \text{ m}^{-3},$$

$$T(s) = T_0 (1 - 0.9999 s).$$

The density and temperature profiles are assumed to be the same for electrons and ions, and the ion mass is equal to the proton mass. The plasma β can be varied by changing the parameter T_0 .

Due to the discrete symmetry of the four field periods, every eigenmode is composed of one of three families of coupled toroidal Fourier harmonics: $\{0, 4, 8, \dots\}$, $\{1, 3, 5, \dots\}$ and $\{2, 6, 10, \dots\}$. When all harmonics up to $n = 31$ are included in the calculation, the fastest growing eigenmode appears in the $\{0, 4, 8, \dots\}$ family, and eigenmodes from the other two families have slightly smaller growth rates. For this reason, and to save computational cost, benchmark calculations use a sector spanning only one field period, *i.e.*, only the $n = 0, 4, 8, \dots$ harmonics are included in the computations.

For the benchmark, four different equilibria are generated by setting $t_0 = 1.02$ and $t_0 = 1.03$ at $\beta = 2.5\%$ and at $\beta = 5\%$. The interchange mode with $(m, n) = (4, 4)$ as its dominant harmonics is considered. We refer to such a mode as the “ (m, n) ” mode due to its dominant harmonics, but it should be noted that the eigenmode is composed of all harmonics included in the calculation.

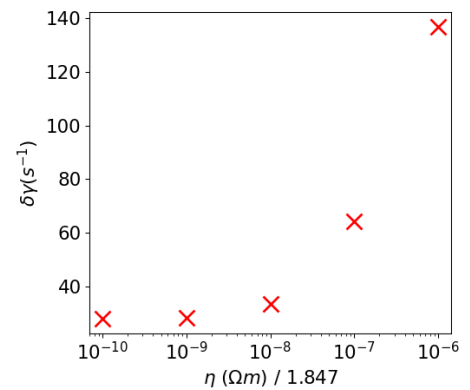


FIG. 10: Growth rate of the (4, 4) interchange mode in the equilibrium with $\beta = 2.5\%$ and $t_0 = 1.02$ is independent of resistivity in the ideal limit. Here, the growth rate variation is shown in terms of $\delta\gamma = \gamma - 6.2 \times 10^4 \text{ s}^{-1}$.

NIMSTELL calculations use 28 radial and 64 poloidal bi-quartic elements in an annular region surrounding a region

of 256 elements in the vicinity of the magnetic axis. The annular mesh is packed near the $t(s) = t_0$ surface. In contrast to Section 3.2, the polynomial degree of the flow-velocity field is equal to that of other fields, including the components of $\mathbf{B}$ in H^1 space, with the computations using the stabilization method described in Refs. 14 and 38 to ensure convergence from the stable side. The time step size is 3.4×10^{-7} s. Figure 10 shows a parameter scan with respect to resistivity in the $\beta = 2.5\%$, $t_0 = 1.02$ case. Since the growth rate is independent of resistivity at low values, the instability is ideal. The resistivity value $\eta = 1.847 \times 10^{-8} \Omega\text{m}$ is used for the rest of this section, as decreasing the resistivity further by an order of magnitude changes the growth rate by only 0.008%. Based on previous benchmarks, the viscosity is set by $\nu_{iso} = 1.47 \times 10^{-5} \text{ m}^2/\text{s}$, which gives the magnetic Prandtl number $Pm = 10^{-3}$. Increasing the viscosity by an order of magnitude changes the growth rate by only $\sim 1 \times 10^{-5}\%$. In CASTOR3D, equilibrium fields are represented with 101 radial grid points, and poloidal and toroidal harmonics up to $m = 60$ and $n = 60$, respectively. The solution fields are represented by Fourier series containing the harmonics $-16 \leq m \leq 16$ and $n = 4k$, $0 \leq k \leq 6$.

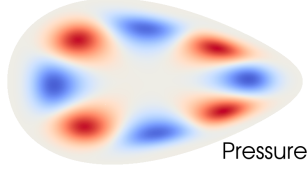


FIG. 11: The (4, 4)-dominant global interchange mode eigenfunction in the $\beta = 2.5\%$, $t = 1.03$ equilibrium is shown in terms of the perturbed pressure in NIMSTELL.

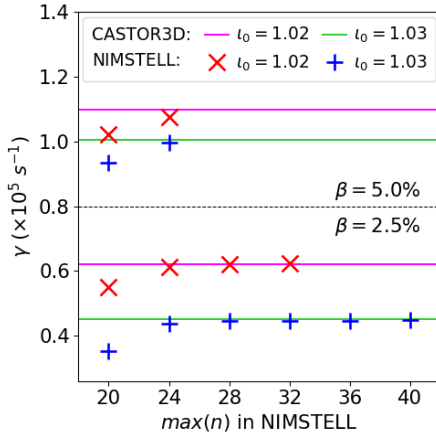


FIG. 12: Growth rates of the (4, 4) interchange mode in four different equilibria are calculated using NIMSTELL and CASTOR3D. The x-axis represents the toroidal resolution in NIMSTELL. For each equilibrium, only those data points are included for which the (4, 4) interchange mode is dominant. The dashed line separates the results for $\beta = 2.5\%$ and $\beta = 5.0\%$ in the bottom and top halves, respectively.

The eigenfunction for the (4, 4)-dominant global interchange mode obtained in the $\beta = 2.5\%$, $t = 1.03$ case is displayed in Figure 11. Figure 12 shows the growth rates computed by CASTOR3D and NIMSTELL, where the NIMSTELL growth rates have been computed using a range of toroidal resolution to show convergence. With the exception of the $\beta = 2.5\%$, $t = 1.03$ case, increasing toroidal resolution eventually allows the (8, 8) interchange mode to grow faster and obscure the (4, 4) mode in NIMSTELL. Truncating the Fourier series renders the (8, 8) mode subdominant in the scans shown here, which allows the (4, 4) mode to be recovered. This truncation has a smaller effect on the growth rate of the (4, 4) mode, as evident from the converging values for the $\beta = 2.5\%$ cases. The CASTOR3D eigenmode computations compute both modes, regardless of which has the largest growth rate.

In both $\beta = 2.5\%$ cases, the growth rates change by less than 0.45% when resolution is increased from $\max(n) = 28$ to $\max(n) = 32$. Thus, we consider these calculations to be well converged. The maximum deviation between NIMSTELL and CASTOR3D growth rates for these cases is 0.95%. Convergence tests for $\beta = 5\%$ prove more challenging because the toroidal resolutions above $\max(n) = 24$ allow the (8, 8) mode to be dominant. Presumably, this is because a larger pressure gradient at $\beta = 5\%$ preferentially destabilizes the (8, 8) mode over the (4, 4) mode. The maximum resolution at which the (4, 4) mode remains dominant is $\max(n) = 24$. Increasing the resolution beyond $\max(n) = 24$ in the $\beta = 2.5\%$ cases changes the growth rate by less than 2.0%. Since we expect the eigenfunction for the (4, 4) mode to be similar for the two β 's, we consider $\max(n) = 24$ to be reasonably accurate for the $\beta = 5\%$ cases. This is further supported by the agreement between NIMSTELL and CASTOR3D growth rates for $\beta = 5\%$, which are within 1.6% of each other.

Two nonlinear computations of the global interchange mode are conducted to demonstrate the nonlinear capability of NIMSTELL and to highlight the potential severity of global interchange modes. Both nonlinear cases use the converged poloidal resolution from linear benchmarks. For computational practicality, the computations are terminated once the imminent loss in pressure is apparent. The first computation uses an equilibrium with a reduced $\beta = 1.5\%$ and $t_0 = 1.02$, because linear tests show better convergence with toroidal resolution at $\beta = 1.5\%$ than at $\beta = 2.5\%$. Only one field period is modeled and toroidal harmonics up to $n = 40$ based on 64 poloidal planes are included. While the restricted toroidal domain cannot capture nonlinear coupling between the mode families $\{0, 4, 8, \dots\}$, $\{1, 3, 5, \dots\}$ and $\{2, 6, 10, \dots\}$, it can represent the early nonlinear phase where the coupling is weak. The dissipative parameters are constant and uniform, and set to $D = 0$, $\eta/\mu_0 = 1.47 \times 10^{-2} \text{ m}^2/\text{s}$, $Pm = \frac{\nu_{iso}}{\eta/\mu_0} = 10^{-3}$, and $\kappa_{||} = \kappa_{\perp} = 0$. The resulting Lundquist number is $S \approx 10^8$. The time step during the linear phase is 6.8×10^{-7} s, and is gradually reduced to 1.7×10^{-7} s in the nonlinear phase to ensure that the algebraic solves converge. Figure 13 shows the development of mushroom-shaped contours of pressure consistent with the expected (4, 4)-dominant interchange mode structure.

The second nonlinear computation models the full torus for

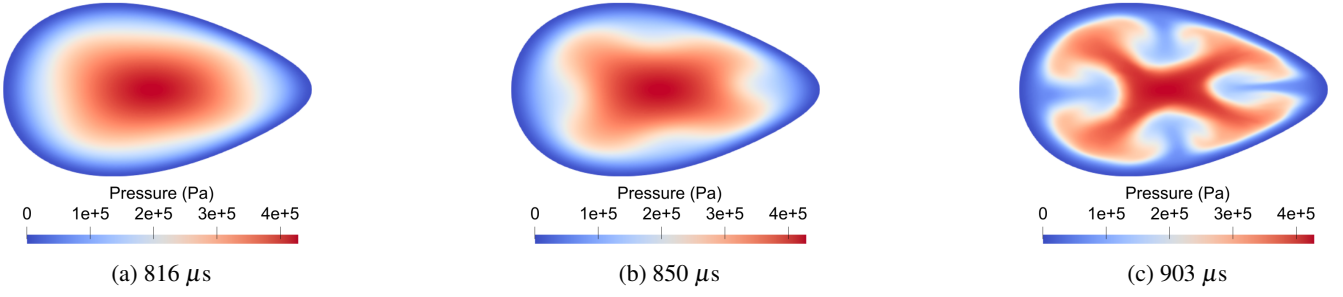


FIG. 13: Nonlinear evolution of the total pressure in the $\beta = 1.5\%$, $t_0 = 1.02$ QH equilibrium is shown in terms of level contours at three time slices. Only one of the four field periods is simulated.

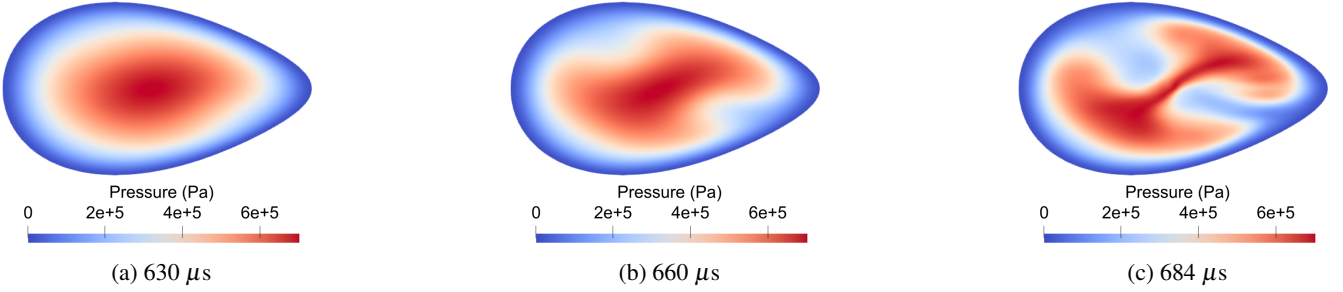


FIG. 14: Nonlinear evolution of the total pressure in the $\beta = 2.5\%$, $t_0 = 1.03$ QH equilibrium is shown in terms of level contours at three time slices. The full torus is simulated in this computation.

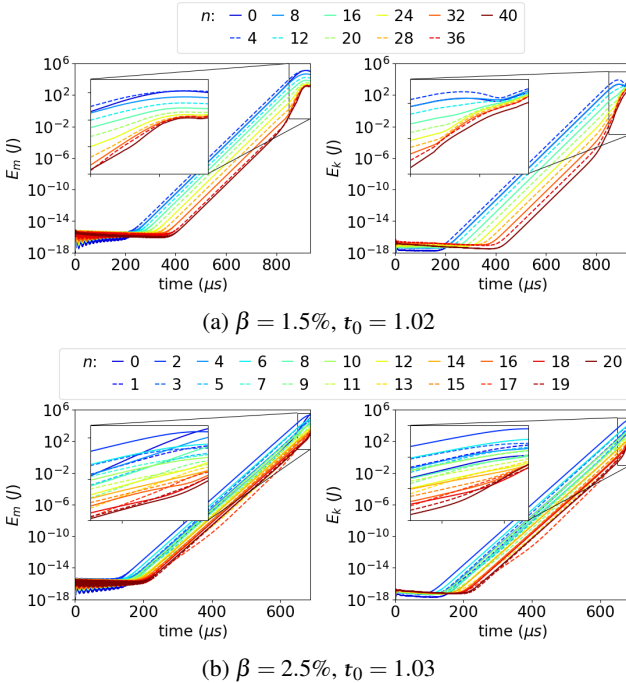


FIG. 15: Temporal evolution of the Fourier components of magnetic and kinetic energies are shown for (a) the $\beta = 1.5\%$, $t_0 = 1.02$ QH equilibrium, where only harmonics that are multiples of four are present, and (b) the $\beta = 2.5\%$, $t_0 = 1.03$ QH equilibrium, where harmonics up to $n = 25$ are present but $n > 20$ are removed for ease of visualization.

the equilibrium with $\beta = 2.5\%$ and $t_0 = 1.03$. Based on the convergence tests shown in Figure 12, toroidal harmonics up to $n = 25$ are included as a compromise between computational cost and accuracy. The number of poloidal planes is 128. The constant and uniform dissipative parameters are set to $D = 10^{-3}$, $\eta/\mu_0 = 1 \times 10^{-2} \text{ m}^2/\text{s}$, $Pm = \frac{v_{iso}}{\eta/\mu_0} = 1 \times 10^{-1}$, and $\frac{\kappa_{||}}{n} = 10^6 \cdot \frac{\kappa_{\perp}}{n} = 1 \times 10^3 \text{ m}^2/\text{s}$. The Lundquist number is $S \approx 10^8$. Here, the time step during the linear phase is $3 \times 10^{-7} \text{ s}$, and is gradually reduced to $1.5 \times 10^{-7} \text{ s}$ in the nonlinear phase. Figure 14 shows a (2, 2)-dominant structure developing in the pressure contours. The behavior is qualitatively similar to the first case despite the inclusion of nonlinear coupling between all mode families. In both cases, the Alfvén time is $\tau_A \approx 1.3 \mu\text{s}$, meaning that the mushroom-like features develop from peaked profiles within a duration of $\sim 50\tau_A$, which is very rapid from an MHD-stability standpoint. Figure 15 shows the Fourier components of the magnetic and kinetic energies as functions of time for both nonlinear cases.

V. DISCUSSION AND CONCLUSIONS

The numerical computations presented in this article verify the NIMSTELL code through benchmarking against CASTOR3D and JOREK for two W7-A applications and a QA application and against CASTOR3D for a QH configuration. In the W7-A cases, NIMSTELL and CASTOR3D growth rates agree to within 1.5% for the tearing mode and within 5.8% for the ballooning mode. The difference between JOREK and

CASTOR3D results for these cases is 11% and 6.2%, respectively. For a tearing mode in the QA configuration, NIMSTELL growth rates agree with CASTOR3D and JOEREK to within 3.0% and 15%, respectively. The agreement with CASTOR3D is within 1.6% for a global interchange mode in the QH cases. In addition to the linear benchmarking, the nonlinear capability of NIMSTELL is demonstrated by modeling the evolution of global interchange modes in two QH equilibria.

We find better agreement between the NIMSTELL and CASTOR3D results in the W7-A cases than with the JOEREK results, which may be attributed to the approximations that are inherent the reduced-MHD model, as discussed in Ref. 16. Additionally, the largest deviation between NIMSTELL and CASTOR3D growth rates is found to be sensitive to viscosity. This suggests that the hyperviscosity in the JOEREK computations may also contribute to quantitative discrepancies. Thus, the difference in viscosity models also cannot be discounted when comparing the time-dependent MHD results, even when using relatively small viscosity values.

As discussed in Section IV B, our W7-A ballooning computations with NIMSTELL use damping to restrict the growth of undesired higher- n ballooning modes, whereas JOEREK and CASTOR3D use a truncation of the Fourier series for the perturbations. Consequently, a fair comparison of growth rates is challenging, as it is for any mode that grows faster with increasing wavenumber. The NIMSTELL results show that components of the ballooning eigenfunction in Fourier harmonics that are higher than the dominant one are important for resolving the eigenfunction. While the NIMSTELL result may be closer to the true eigenfunction shape, the use of artificial damping to converge on the $n_{dom} = 10$ mode may limit the accuracy of the result, nonetheless.

For ideal ballooning modes, the growth rate continues to increase with decreasing wavelength as the wavelength approaches zero. However, with nonzero viscosity, modes with wavelengths shorter than a threshold can be rendered subdominant by viscous dissipation. Thus, a fair comparison between codes may be possible by modeling ballooning modes with a suitable combination of viscosity and resolution. A more realistic alternative is to suppress the growth of shorter-scale ballooning modes by including equilibrium flow or finite-Larmor-radius effects. However, modeling of these effects is beyond the scope of this work.

The agreement between NIMSTELL, CASTOR3D and JOEREK for the tearing mode in QA verifies the respective numerical implementations for resistive dynamics in highly shaped configurations. The lowest resistivity case ($S \sim 10^8$) demonstrates that approaching the asymptotic behavior is possible in NIMSTELL and JOEREK, which may be important for simulating experimentally relevant regimes. Moreover, this benchmark serves as a starting point for nonlinear computations of tearing mode saturation in a QA configuration.¹⁴

The QA tearing computations also highlight the possible consequences of having even small residual force errors in the initial equilibrium. They can modify the initial plasma state in nonlinear computations and affect the subsequent linear growth of instabilities. The discrepancy in our linear study is mitigated by freezing the even toroidal harmonics in JOEREK,

which is only possible due to the decoupling of the linear instability from equilibrium evolution. However, careful treatment of equilibrium force error will be necessary for accurate linear and nonlinear modeling in other scenarios.

Along with the W7-A ballooning application, the QH-interchange benchmark tests pressure-driven dynamics, which is a significant focus in the MHD stability analysis of stellarators. In our QH cases, the magnetic shear is vanishingly small in a region where the rotational transform is close to a low-order rational number. Consequently, growth rates of low- n interchange modes can be greater than those of the large- n ballooning modes, making it possible to resolve low- n modes without damping higher toroidal harmonics. Contrast this with the W7-A ballooning case, where a non-physical damping term of high harmonics is used to prevent faster-growing ballooning modes from obscuring the mode of interest. While convergence tests for the interchange mode must also be treated carefully, as discussed in section IV D, they prove to be more tractable for quantitative verification than with the ballooning modes. In addition to demonstrating code capability, the nonlinear computations of the non-resonant global interchange modes suggest that they can lead to rapid and substantial loss of pressure. It would, therefore, be prudent to consider their stability in stellarator optimization studies. Further investigation of their dynamics is warranted and may lead to a new stability proxy that is suitable when scanning many configurations during optimization.

Collectively, the benchmarks presented here verify NIMSTELL's implementation of semi-implicit, spectral-element MHD. A comprehensive set of cases is developed that spans dynamics relevant to time-dependent modeling of modern stellarator configurations, namely, tearing, ballooning, and interchange instabilities. The use of representative geometries informs baseline expectations for the spatial and temporal resolutions necessary for simulating optimized configurations. While the presented computations may have limited physical relevance due to the tailored nature of the equilibria, this work establishes guidelines and forms the basis for future modeling of linear and nonlinear physics with NIMSTELL.

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at the Max Planck Computing and Data Facility (MPCDF),
and the HPC system Pitagora, hosted by CINECA (Italy).

Appendix A: JOREK's reduced-MHD model

The JOREK computations used for benchmarking in this work use the simplest Ansatz-based reduced MHD of Refs. 36 and 16. This model does not include the perpendicular compression terms or the parallel flow equation that is now available in JOREK.³⁴ In this reduced form, the magnetic field and flow-velocity vectors are:

$$\mathbf{B} = \nabla\chi + \nabla\Psi \times \nabla\chi, \quad (\text{A1})$$

$$\mathbf{V} = B_v^{-2} \nabla\Phi \times \nabla\chi, \quad (\text{A2})$$

where χ is a potential for the stationary vacuum magnetic field ($\mathbf{B}_v = \nabla\chi$), Ψ is a flux function for the time-dependent field, and Φ is the time-dependent stream function. Differential operators are expressed relative to total and vacuum fields:

$$\begin{aligned} \partial^\parallel f &= B_v^{-1} \nabla\chi \cdot \nabla f, \\ \nabla^\perp f &= \nabla f - B_v^{-1} \nabla\chi \partial^\parallel f, \\ \Delta^\perp f &= \nabla \cdot \nabla^\perp f, \\ \Delta^* f &= B_v^{-2} \nabla \cdot (B_v^2 \nabla^\perp f), \\ \nabla_\parallel f &= B^{-2} \mathbf{B} \cdot \nabla f, \text{ and} \\ \nabla_\perp f &= \nabla f - \nabla_\parallel f. \end{aligned}$$

$$\frac{\partial \rho}{\partial t} = -B_v \left[\frac{\rho}{B_v^2}, \Phi \right] + P, \quad (\text{A3})$$

$$\begin{aligned} \nabla \cdot \left(\frac{\rho}{B_v^2} \nabla^\perp \frac{\partial \Phi}{\partial t} \right) &= \frac{B_v}{2} \left[\frac{\rho}{B_v^2}, \frac{(\Phi, \Phi)}{B_v^2} \right] + B_v \left[\frac{\rho \tilde{\omega}}{B_v^4}, \Phi \right] - \nabla \cdot \left(\frac{P}{B_v^2} \nabla^\perp \Phi \right), \\ &+ \nabla \cdot (\tilde{j} \mathbf{B}) + B_v \left[\frac{1}{B_v^2}, p \right] + \nabla \cdot (\mu_\perp \nabla^\perp \tilde{\omega}) - \Delta^\perp (\mu_h \Delta^\perp \tilde{\omega}), \end{aligned} \quad (\text{A4})$$

$$\begin{aligned} \frac{\partial p}{\partial t} &= -\frac{1}{B_v} [p, \Phi] - \gamma p B_v \left[\frac{1}{B_v^2}, \Phi \right] \\ &+ \nabla \cdot \left[(\gamma - 1) \kappa_\perp \nabla_\perp T + (\gamma - 1) \kappa_\parallel \nabla_\parallel T + \frac{p D_\perp}{\rho} \nabla_\perp \rho + \frac{p D_\parallel}{\rho} \nabla_\parallel \rho \right] \\ &+ (\gamma - 1) (S_e + \eta B_v^2 \tilde{j}^2), \text{ and} \end{aligned} \quad (\text{A5})$$

$$\frac{\partial \Psi}{\partial t} = \frac{\partial^\parallel \Phi - [\Psi, \Phi]}{B_v} - \eta (\tilde{j} - \tilde{j}_0) + \nabla \cdot (\eta_h \nabla^\perp \tilde{j}). \quad (\text{A6})$$

The auxiliary variables for current density and vorticity are related to the flux function and stream function via $\tilde{j} = \Delta^* \Psi$ and $\tilde{\omega} = \Delta^\perp \Phi$. The notation includes $P = \nabla \cdot (D_\perp \nabla_\perp \rho + D_\parallel \nabla_\parallel \rho) + S_\rho$. The equations include source densities for mass (S_ρ), energy (S_e), and current ($\eta \tilde{j}_0$) to preclude diffusion of a prescribed equilibrium without the separate representation of steady-state fields that is used in NIM-STEEL. The coefficient η_h is the hyper-resistivity.

The Poisson bracket $[f, g] = B_v^{-1} \nabla\chi \cdot (\nabla f \times \nabla g)$ and the inner product $(f, g) = (\nabla^\perp f \cdot \nabla^\perp g)$ are used. With these definitions, the evolution equations for the mass density, stream function, pressure p , and flux function are:

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