Sloshing Ion Stabilization of Mirrors and Cusps

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Abstract

A symmetric mirror or cusp can be stabilized by sloshing ions. Sloshing ions can stabilize DCLC that has limited performance in mirrors to date. Sloshing ions can also stabilize MHD. Sloshing ion stabilization of both DCLC and MHD can be demonstrated in the high-field mirror facility at the University of Wisconsin, Madison. We give examples of mirror reactors stabilized by sloshing ions, including a DT experiment producing net electric power, in a device similar in size to the Madison mirror facility. Magnetically stable mirror-cusps with DCLC stabilized by sloshing ions are competitive with the ARC tokamak pilot plant. Simple mirror reactors can be even smaller and cheaper if MHD can be stabilized some other way.

I. Fusion Power Gain Q

Optimally, the fusion power gain Q in a mirror device is determined only by end losses for scattering time τ_{ii} [1]:

$$Q = 1/4\ n\ [\tau_{ii}/P(R_M)](E_{DT}/E_i) \quad (1)$$

where $P(R_M)$ is the fraction of ions in the loss cone. Typically the optimum E_i = 100 – 200 keV, by NBI, but we are free to choose a lower beam energy augmented by ICH if that is a better match to 100% beam deposition. Validity requires stability, and radial losses less than the unavoidable end losses.

$P(R_M)$ is key. Kaufman estimated $P = [1 - (1 - (1/R)^{1/2})]$ [2]. In Sect. IV, we use $P = 2 \log_{10} R$ [1,3]. Pastukhov gave an exponential factor for electrons confined by a potential [1,4]. Such estimates have been applied to isotropic Fokker-Planck codes (function of energy E only) [5]. None of these estimates is likely to be accurate for the beta-enhanced mirror ratios in Sect. IV and App. A, for which a 3D Fokker-Planck (E, $v_{||}$, z) may be required. Calibrating improved theoretical calculations of

$P(R_M)$ in the 12:1 vacuum mirror-ratio device at Madison could demonstrate the stability of mirrors and the ability to calculate mirror performance from collisions alone [4].

II. Sloshing Ion Stabilization

By sloshing ions we mean neutral beams aimed to cause ions to bounce between mirrors [6]. We are free to optimize the neutral beam energy to achieve 100% beam deposition, with ion energy augmented by ICH.

All mirror experiments to date have been limited by DCLC, an exception being BBII that was completely stable ($\tau \approx 1$s) at a density too low to excite DCLC [7]. While DCLC is predicted to be stable in a sufficiently large device [1,4], a low-cost development path requires sloshing ions, both for DCLC and AIC modes [1,4]. To stabilize DCLC, the density at the sloshing-ion turning points n_{SI} must equal or exceed the midplane density n of ions accumulated by scattering ($n_{SI} \geq n$). Then the plasma potential drop causing DCLC is largely pushed beyond the mirror throat. Stabilizing MHD is more restrictive, requiring $n_{SI}V_{SI}/R_{Ci} \geq nV_R/R_{CR}$ for volume V_{SI} around the sloshing ion density peak and V_R in the region of bad curvature, with curvature radii R_{Ci} and R_{CR} in good and bad regions, ratio $F_C = R_{Ci}/R_{CR}$. Since $V_{SI} < V_R$, then $n_{SI} > n$ which stabilizes DCLC also. Ref. [8] argues that, for a single mirror, MHD stabilization also stabilizes trapped particle modes. Sloshing electrons (rather than sloshing ions) are discussed in Ref. [9].

In a configuration consisting of a solenoid of length L with either symmetric mirror-cusps [10] or simple mirrors at each end, fusion is produced in a volume $V \geq V_R(L/R)$. To avoid excess particle feed, NBI producing sloshing ions must satisfy:

$$2(n_{SI}V_{SI}/\tau_{ii}) \leq nV(P/\tau_{ii}) = nV_R(L/R)(P/\tau_{ii}) \quad \text{Density limit} \quad (2a)$$

$$L/R \geq 2\,[(n_{SI}V_{SI})/nV_R](1/P) \quad (2b)$$

$$L/R \geq (2F_C/P) \quad \text{MHD} \quad ; \quad L/R \geq 2[(V_{SI}/V_R)(1/P)] \rightarrow 1 \quad \text{DCLC} \quad (2c)$$

Also, the sloshing ion beta limits the sloshing ion density. In terms of midplane beta β_0 and field B_0, and guessing $B(R_{SI}) \leq ½ B_M$, we obtain:

$$n_{SI}/n \quad \leq \quad (1/2\beta_0)(B_M/B_0)^2 \qquad \text{Beta limit} \qquad (3)$$

Mirror-cusp coils are larger than simple mirrors, hence more expensive; but, being MHD stable, they offer more freedom to heat ions by ICH if sloshing ions are only needed for DCLC/AIC. Design optimization could trade higher cost cusp magnets versus lower NBI/ICH costs due to beta-enhanced mirror ratio. Evidence to date favors higher beta with magnetic stabilization (2XIIB yin-yang, $\beta = 1$ [7]; axi-symmetric GDT, $\beta = 0.6$ [11]). We will return to these points in Appendix A.4. Other ways to stabilize MHD are discussed in [6].

The Madison mirror facility with large L/R can explore sloshing ion stabilization for any path forward using mirrors or mirror-cusp geometry.

III. Radial Losses

As beta decreases the field at r = 0, we require that a DT orbit barely fit inside the wall radius (similar to cusp analysis by Grosssman [12], who showed that local B = 0 defines a wall containing unconfined plasma). At $\beta = 1$ with midplane vacuum field B_0 giving a mirror ratio R_M, the effective Larmor radius is $R_L = R_{LO} /(1 - \beta_1)^{1/2} <$ R. This defines β_1, producing an effective mirror ratio R_M given by:

$$R_M = [(B_M/B_0(1 - \beta_1)^{1/2}] = (B_M/B_0)(R/R_{LO}) \qquad (4)$$

with wall radius R and midplane field B_0. Both ion collisions and the ETG turbulence of tokamaks can produce radial loss in mirrors. At $\beta = 1$ the field is confined to a pedestal of width $\Delta_{mag} = (R_m^2/R)$ with plasma radius R_m at the mirror throat [4, Section 7.5, with error omitting factor B_C in Eq. (26c)].

Other relevant quantities determining radial confinement are (SI units, E in keV, $n_{20} = n/10^{20}$, DT plasma, using Eq. (4)):

τ_{mirror}	=	$(1.6 \times 10^{-4} E^{3/2}/n_{20}) \log_{10} [(B_M/B_0)(R/R_{LO})]$	(5a)
χ_{ii}	=	$(n_{20}/E^{1/2}B^2)$	(5b)
χ_{ETG}	=	$0.2\ (E^{3/2}/B^2R)$	(5c)
$1/\tau$	=	$(1/\tau_{mirror}) + (\chi_{ii} + \chi_{ETG})/\Delta_{mag}^2$	(5d)
Q	=	$1/4\ (n\tau)(\sigma v)_{DT}\ [E_{DT}/[<E_i> + (\phi + T_e)]$	(5e)

For a sufficiently short length L between mirrors, ETG can be prevented by Landau damping [7], perhaps establishing a maximum L/R in Eq. (2b). As noted above, at high beta the mirror magnetic flux ψ is pushed to the wall giving a width $\Delta = (\psi/2\pi R_{WALL}\ B_0)$. For this the criterion for Q in Eq. (1) to prevail is $[(\Delta^2/\chi_{ii}) + (\Delta/r_L)^2\tau_{ii}] > (\tau_{ii}/P)$ with Larmor radius r_L.

IV. Reactor Prospects; A Fusion Chicago Pile

The following examples illustrate the benefit from large R_M. We show that the smallest mirror device burning DT could have a plasma radius of order 0.05m. This is similar in scale to 2XIIB and the Madison mirror facility. To illustrate a hierarchy of scales, culminating in an attractive power plant, we apply the following design criteria for a magnetically-stabilized spherical volume, assuming DCLC stability also; B_M = 17 T at the mirror throat; and beta-enhanced mirror ratio R_M:

(a) From Eq. (4), with $r_{Lo}(100\ \text{keV}) = (0.07\ \text{m}/B_0\sqrt{(1 - \beta)})$, we obtain:

$$R_M = (B_M/B_0)(R/r_{Lo}) = (B_M R/0.07) \quad (6)$$

(b) Calculate power gain Q scaled from results in [5]:

$$Q = P(R_M) \approx 2 \log_{10} R_M \quad (7)$$

(c) Choose neutron wall load 2.5 MW/m² at R, giving DT power:

$$P_{DT} = 4\pi R^2\ (2.5)\ \text{MW} \quad \text{quasi-spherical} \quad (8)$$

Conditions (a), (b) and (c) determine R_M, Q and fusion power P_{DT} given the plasma (wall) radius R, with results in the table below:

Symmetric (Mirror-Cusp) Reactor Parameters

	R	R_M	Q	P_{DT}	Q_{eff}	P-electric
Case 1	0.05 m	12	2.2	50 kW		5 kWe
Case 2	5 m	1200	6.5	785 MW	19	350 MWe

Here $Q_{eff} = Q/[1 - 0.1Q]$ accounts for 50% of alpha power heating the plasma (Case 2 only). To calculate electric power from Q_{eff} requires determining the density n yielding a 2.5 MW/m^2 wall load, giving then requirements on NBI to sustain the reaction, and also the midplane field $B_0 = (8nE_i/\beta_0)^{1/2}$ (typically < 2T). It works out that 20 keV beams are optimum (100% deposition, with ICH boost to 100 keV).

Case 1 is the fusion "Chicago Pile" proof-of-principle, with $R_M = 12$ (for log scaling) and dimensions confining D-T radially (end loss dominant) but too small to contain alphas (Larmor radii $r_\alpha = (0.27/B_0)$; requiring wall protection). For this case $L/R \approx 2$ gives $Q \approx 1$ even for sloshing ion stabilization of MHD. Other ways to stabilize MHD are discussed in [6]. The electric power is the residual from thermal conversion at 40% minus NBI/ICH power = P_{DT}/Q generated at 80 % efficiency and direct conversion efficiency 50% [4, App. B]. The fusion power is 50 kW producing net 5 kWe electric power. The fission Chicago Pile power was 0.5 watt.

Case 2 has $R_M = 1200$ for log scaling. In Appendix A, we discuss how a device of similar size extrapolates to a pilot power plant, with direct conversion in expanders required in any case to achieve high T_e [4]. For Case 2, we include a factor 1.3 blanket multiplication. About 50% of the alpha power heats the plasma, giving $Q_{eff} = Q/[1 - 0.5 \times 0.2 \times Q]$; and from this, a plant efficiency of 45% (fusion to electric).

Applied to mirror-cusp geometry, parameters in the table above need not satisfy Eqs. (3 a,b,c). Interpreting Case 2 as a simple axisymmetric mirror MHD-stabilized by sloshing ions gives P = 0.16 (to give Q = 6.5) which requires L/R =12.5 to satisfy Eq. (3a). Effectively, L/R = 2 for the sphere; hence for the simple mirror P-electric = (12. 5/2)350 = 2000 MWe.

For both mirror-cusp and simple mirror, the midplane field $B_0 \leq 1$ T (to satisfy Eq. (3c); also satisfying Eq. (3b)). A field B_0 = 1T and R = 5m confines DT and alphas ($(r_L\, \alpha)$ = $(0.25/\sqrt{(1 - \beta)} < 5$, $\beta < [1 - (0.25/5)^2] = 0.998$).

Appendix A

A1. Comparing a Mirror-Cusp Reactor to the ARC Tokamak

Here we compare the ARC tokamak pilot plant [13] with a mirror-cusp [10] of equal electric power output (somewhat less than Case 2, Section IV).

Parameters	Mirror	ARC [8]
R_{WALL}/Major Radius	3.3 m (equiv. sphere)	3.3 m
Length between mirrors	10 m	
Plasma volume	150 m^3	141 m^3
Density	10^{20} m^{-3}	10^{20} m^{-3}
Ion energy	100 keV	15 keV (avg.)
B	2T (midplane)	9 T (on axis)
B_{MIRROR}	15.4 T	
Max. B at conductor	23 T	23 T
Injection power	93 MW (D.C. NBI)	70 MW (Ignitor)
P_{FUSION} (MW)	465 MW ($<\beta^2>= 0.5$)	525 MW
$P_{ELECTRIC}$	0.4 x 465 = 190 MW_e	190 MW_e

A.2 Power-Related Costs

We apply the ARC Flibe blanket for the mirror, cost proportional to Fusion power [13]. The internal divertor in ARC is replaced by magnetic flux escaping from the ends of mirrors, providing a natural divertor, shown in figures in Ref. [4].

Power-related Costs	Mirror	ARC
Blanket	$230 M	$260 M
Beam/Ignitor ($10/watt)	$930 M	$700 M
Vessel, Direct converter	$180 M*	$ 92 M
Power Subtotal	$ 1340 M	$1052 M
$P_{ELECTRIC}$	190 MW_e	190 MW_e
Subtotal Capital Cost	$7053/kWe	$5536/kWe

*Taken equal to MFTF-B 72 m-long vessel: $52M plus inflation.

A.3 Total Fusion Reactor Costs

Magnet costs include conductor and structure to contain the forces. We use ARC fabricated cost at $1M/tonne [13]:

Magnet Costs	Mirror	ARC
Magnets	$ 387 M	$ 380 M
Magnet structure	$1900 M	$5100 M
Magnet Subtotal	$2287 M	$5480 M
Power related	$1340 M	$1052 M
Fusion Reactor Total	$3627 M	$6532 M
Turbine Hall (@ $1/watt)	$ 190 M	$ 190 M
Total Power Plant	$3817 M	$6722 M
Net Electric	190 MWe	190 MWe
Total Capital Cost	$20,089/kWe	$35,379/kWE

Coal equivalent (capital plus fuel) $12,000/kWe

A.4. Development Path, Ways to Reduce Capital Cost

Numbers above are for a β = 1, fully-minimum-B stable mirror-cusp configuration such as that in Ref. [10]. Cusp coil radii are order of the wall radius. Magnet cost $\propto R_{MIRR}^2$ for mirror length $L \propto R_{MIRR}$. Then magnet cost is roughly proportional to R_{MIRR}^2, giving a total capital cost $11,200/kWe (like coal) for a factor 2 reduction in R_{MIRR}, given some means of stabilization other than sloshing ions [6], if also beta were still unity (see note end of Sect II). The other area for cost reduction is steady state neutral beams [14], priced here at $10/watt. The total neutral beam cost is driven by Q, hence by β through the beta-enhanced mirror ratio.

Pursuing a DT pilot plant via the single mirror route is also a path toward advanced fuels in tandem mirrors [4], and fusion nuclear engineering development complementing the large ITER tokamak.

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