Toroidal Geometry Effects in the Low Aspect Ratio RFP

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Objective

Determine the influence of toroidal geometry on low-β reversed-field pinch configurations.

Outline

I. Introduction
   A. Background
   B. Geometric considerations
   C. Modeling
II. Aspect ratio scans
   A. F-Θ
   B. Magnetic fluctuations
   C. Spectrum width
III. Laminar RFP states
   A. Single helicity conditions
   B. Quasi-single helicity conditions
IV. Remaining questions
V. Conclusions

Background

- Most numerical simulations and analytic computations for the RFP have been performed in periodic linear geometry.

- This is usually a good approximation.

  - Since $q<1$, pressure gradients cannot stabilize tearing modes (assuming p decreases with r). [Glasser, Greene, and Johnson, Phys. Fluids **18**, 875 (1975).]

  - Strong nonlinear coupling among resonant fluctuations of different poloidal index m is a characteristic of standard RFP operation. [Ho and Craddock, Phys. Fluids B **3**, 721 (1991).]

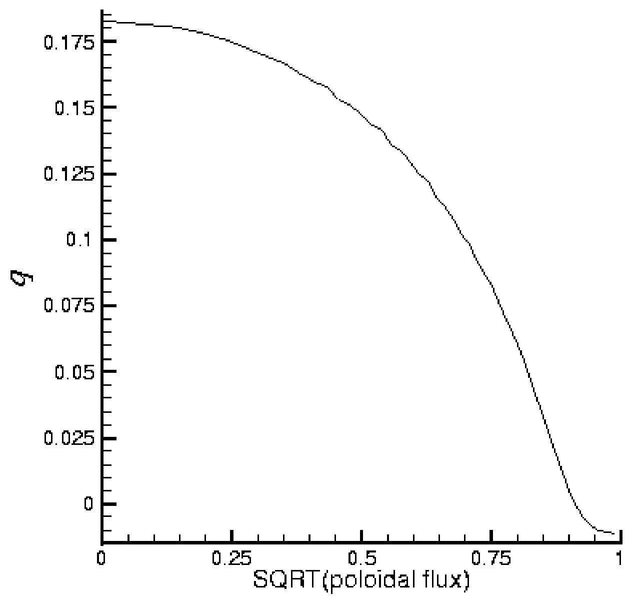
- A laminar version of the RFP dynamo, known as the "single-helicity state," exists at sufficient dissipation levels in periodic linear geometry. [Finn, Nebel, and Bathke, Phys. Fluids B **4**, 1262 (1992); Cappello and Escande, PRL **85**, 3838 (2000).]

  - Usual nonlinear coupling among different helicities is absent.

  - Toroidal geometry effects can make a qualitative difference in these conditions, due to linear coupling of different m.

Geometric Considerations

- Many low-order helicities are resonant in a typical RFP *q*-profile.



- Close spacing of the rational surfaces and the global nature of the dominant tearing modes allow for strong nonlinear coupling in standard operation.

- Well known for tokamaks, toroidal geometry leads to **linear** coupling among helicities of different *m*.

  - The gradient operator contains

$$\frac{1}{R}\frac{\partial}{\partial\varphi} = \frac{1}{(R_0 + r\cos(\theta))}\frac{\partial}{\partial\varphi}$$
$$= \frac{1}{R_0}\left(1 - \varepsilon\frac{r}{a}\cos(\theta) + \left(\varepsilon\frac{r}{a}\right)^2 \cos^2(\theta) - \ldots\right)\frac{\partial}{\partial\varphi}$$

    where $\varepsilon = a/R_0$ and the $\cos(\theta)$ terms lead to the coupling.

  - The Shafranov shift introduces poloidal asymmetries in the equilibrium, also leading to linear coupling.

- For an RFP *q*-profile we can expect the strongest poloidal coupling to occur between *m*=0 and *m*=1 helicities.

Modeling

To investigate the electromagnetic activity, we solve the zero-β resistive MHD equations in circular cross-section, toroidal and periodic linear geometries using the NIMROD simulation code, http://nimrodteam.org.

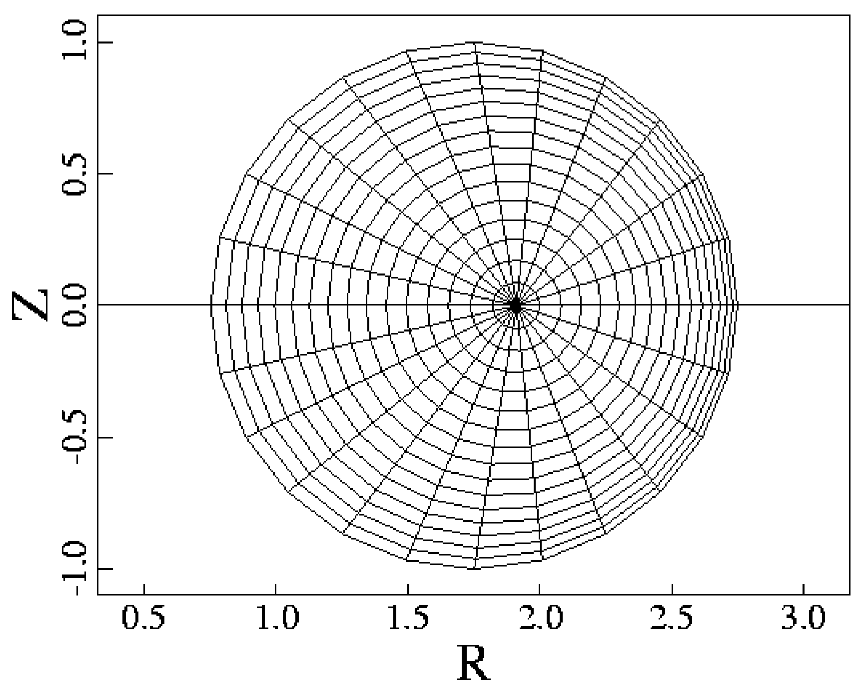
$$\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} = \frac{1}{\rho} \mathbf{J} \times \mathbf{B} + \nabla \cdot \left(\nu \nabla \mathbf{V} \right)$$

$$\mathbf{E} = -\mathbf{V} \times \mathbf{B} + \eta \mathbf{J}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

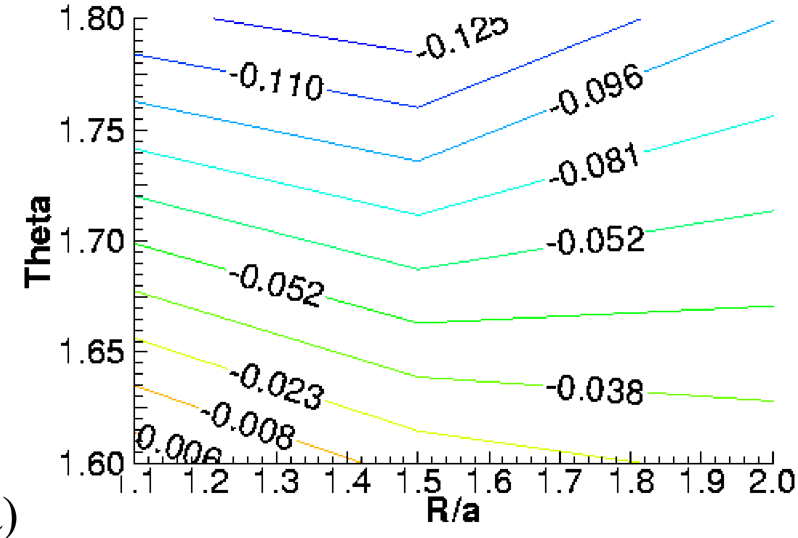
- Density is uniform, though flow is not incompressible.
- Resistivity and viscosity are essentially uniform. [S=2500 and $P_m \equiv \mu_0 \nu / \eta = 1 - 100$]
- Voltage is adjusted to maintain the desired plasma current. The time-scale for the feedback is comparable to the tearing time to avoid excitation of surface currents.
- Two-fluid effects may be important.
  - The drift ordering is more realistic for RFPs than the MHD ordering even at small β.
  - Worth further investigation.
  - Two-fluid capabilities in NIMROD are being developed through the PSACI project.

- Numerical parameters:
  - Most of the simulations reported here have $0 \le n \le 42$.
  - Some of the simulations for laminar conditions have $0 \le n \le 21$.
  - NIMROD uses finite elements to represent the poloidal plane. Simulations for the aspect ratio scan were run with a 48x48 or 64x64 (radial x azimuthal) mesh of bilinear finite elements.
  - Where viscosity is scanned to suppress nonlinear activity, a 16x24 or 16x32 mesh of bicubic elements is used for a better representation of the magnetic field. [See "Nonlinear Fusion Magneto-Hydrodynamics with Finite Elements," Sherwood 2000, in http://nimrodteam.org/presentations.]

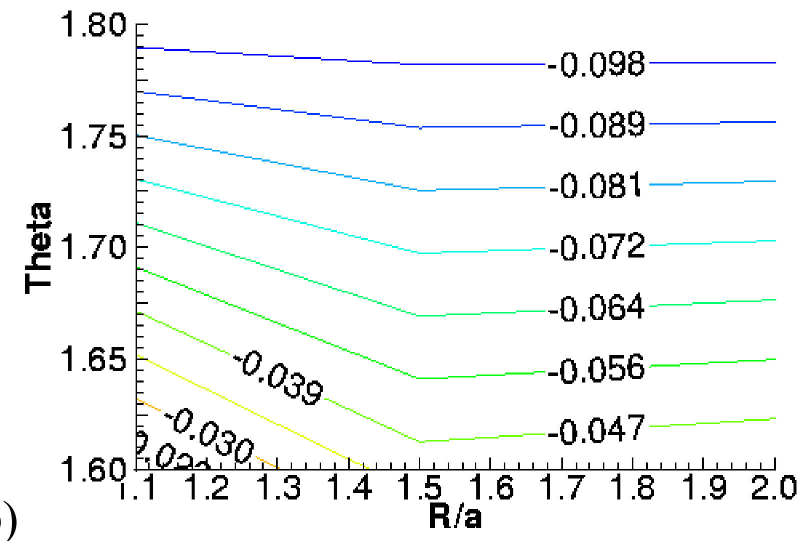


Aspect Ratio Scans in Toroidal and Periodic Linear Geometries

- Results on field reversal from dynamo action are similar in the two geometries, even at very low aspect ratio.



a)



(b)

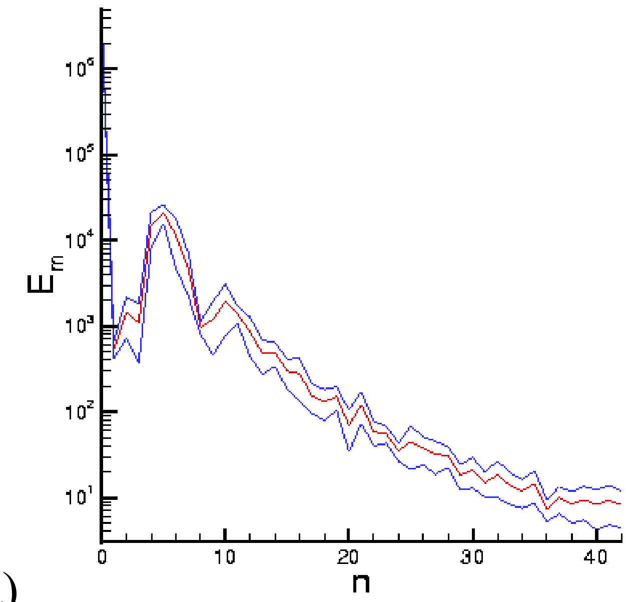
Comparison of time-averaged reversal parameter (F) resulting from simulations in (a) toroidal geometry and (b) periodic linear geometry at S=2500 and P_m=1.

- Magnetic fluctuation levels are also comparable.

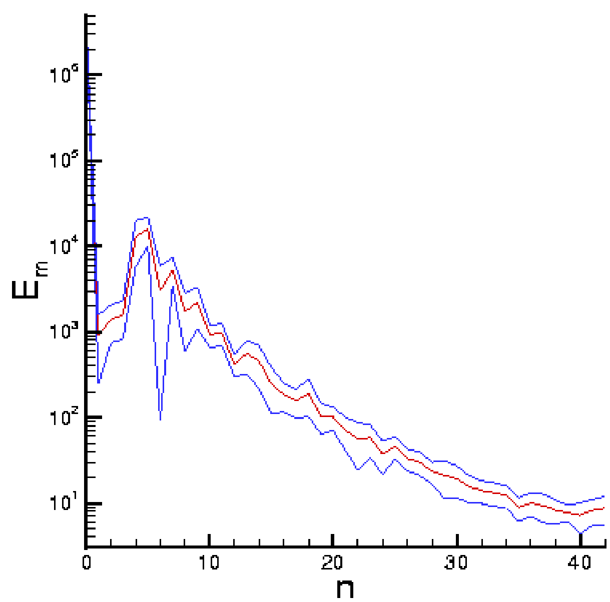
geometry	R/a	Θ	$\frac{n>0\,\text{energy}}{\text{total}\,\text{energy}}$
toroidal	1.1	1.6	0.091
linear	1.1	1.6	0.080
toroidal	1.1	1.8	0.16
linear	1.1	1.8	0.097
toroidal	1.5	1.6	0.083
linear	1.5	1.6	0.085
toroidal	1.5	1.8	0.14
linear	1.5	1.8	0.10
toroidal	2	1.6	0.084
linear	2	1.6	0.087
toroidal	2	1.8	0.10
linear	2	1.8	0.10

Results are averaged over 1-2 tenths of a global diffusion time.

- Magnetic energy spectra plotted vs. *n* and summed over *m* for the two geometries are often nearly indistinguishable for standard multi-helicity states.



a)

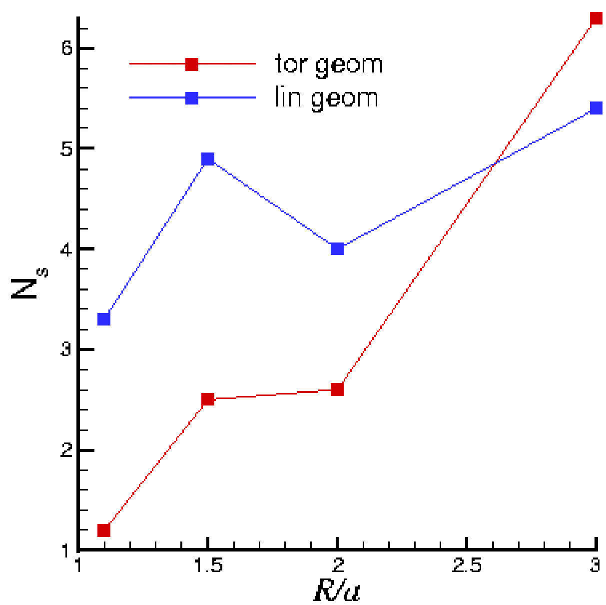


b)

Magnetic fluctuation energy spectra for a) toroidal geometry and b) periodic linear geometry showing the temporal average (red) and ± one standard deviation (blue) for R/a=1.75, P_m=1, Θ=1.8.

- The spreading of the magnetic spectrum with *R/a* reported by Ho, *et al.* ["Effect of aspect ratio on magnetic field fluctuations in the reversed-field pinch," Phys. Plasmas **2**, 3407 (1995).], is also observed in toroidal geometry.

  - Each W_n is summed over m.
  - Nonlinear interaction seems to be more easily suppressed in toroidal geometry.



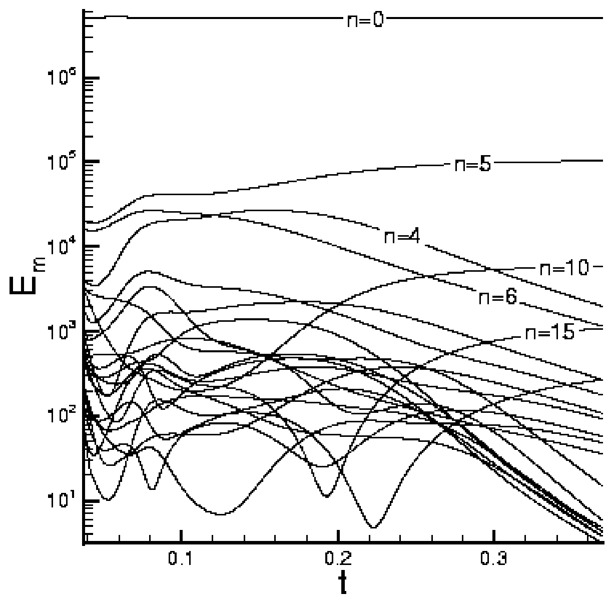
Simulation results on $N_s \equiv \left(\sum_n W_n\right)^2 \Big/ \sum_n W_n^2$ for Θ=1.6, P_m=1 simulations. At *R/a*=1.5, *q*(0) is slightly greater than 1/3.

Laminar RFP States

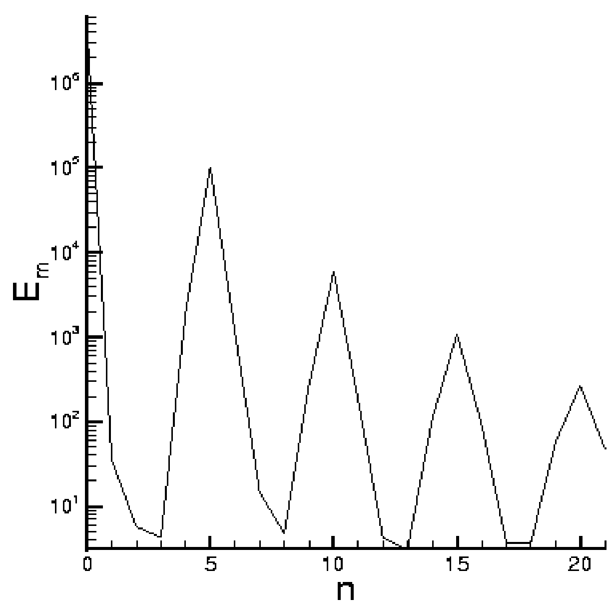
- As viscosity is increased there is a transition to steady or near-steady states.
- Cappello and Escande have established that this transition is more dependent on the Hartmann number ($H \equiv SP_m^{-1/2}$) than the Lundquist number.

Transition to laminar states in periodic linear geometry with *R/a*=4. [Cappello and Escande, "Bifurcation in Viscoresistive MHD: The Hartmann Number and the Reversed Field Pinch," PRL **85**, 3838 (2000).]

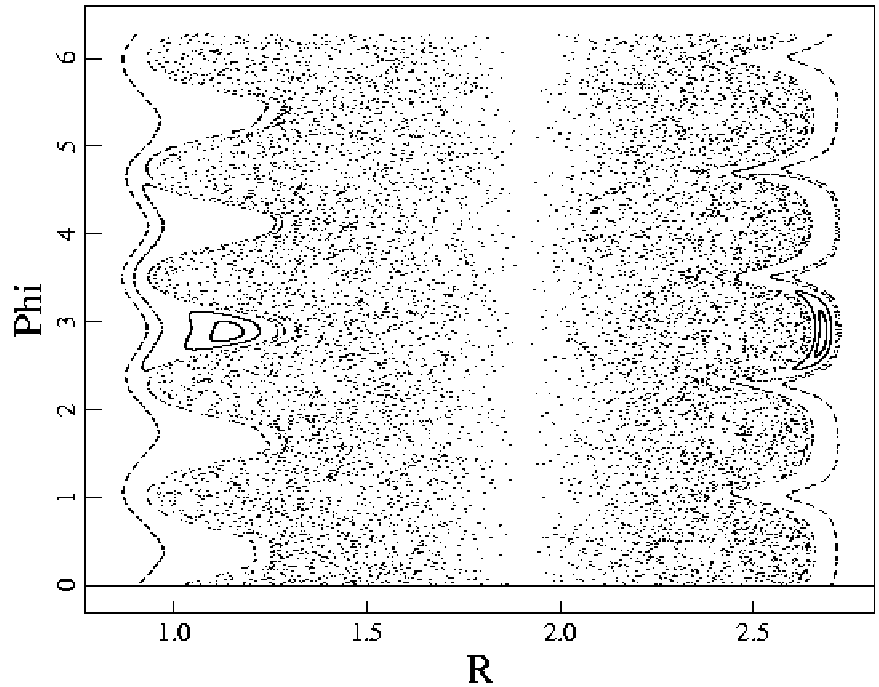
- A transition to laminar behavior also occurs in toroidal geometry as P_m is increased. The following figure shows the transition in the toroidal R/a=1.75, Θ=1.8 case after P_m is increased from 1 to 10.



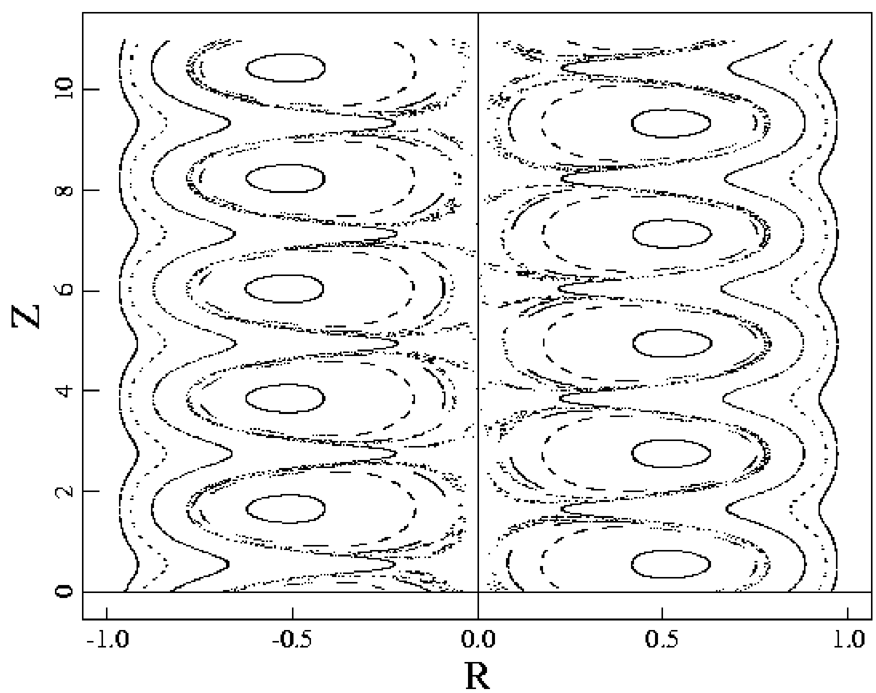
- Plotted vs. n and summing over m, the spectrum has the appearance of a single helicity state ($E_{(m,n)}$=0 if $m/n \neq 1/n_p$, where n_p is the toroidal index at the peak of the spectrum).



- Poincaré surfaces of section for **B** show that the final state in toroidal geometry is not single-helicity, however.



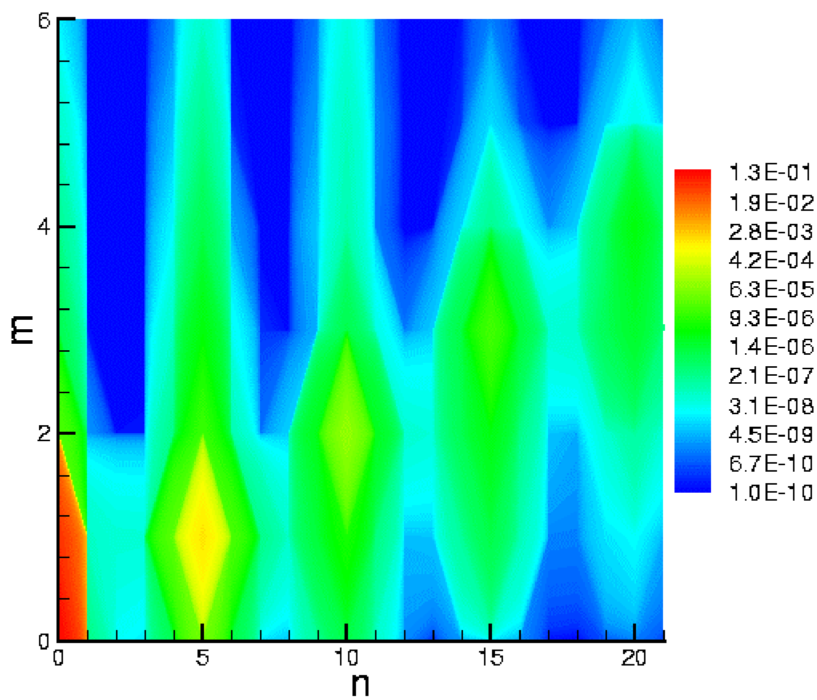
a)



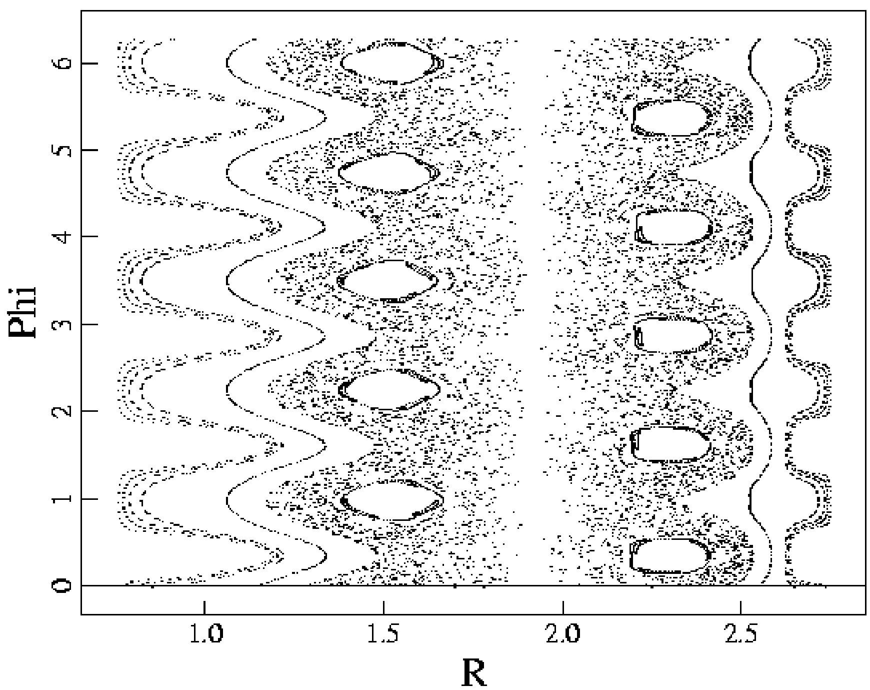
b)

Results from a) toroidal geometry and b) periodic linear geometry with P_m=10, R/a=1.75, Θ=1.8.

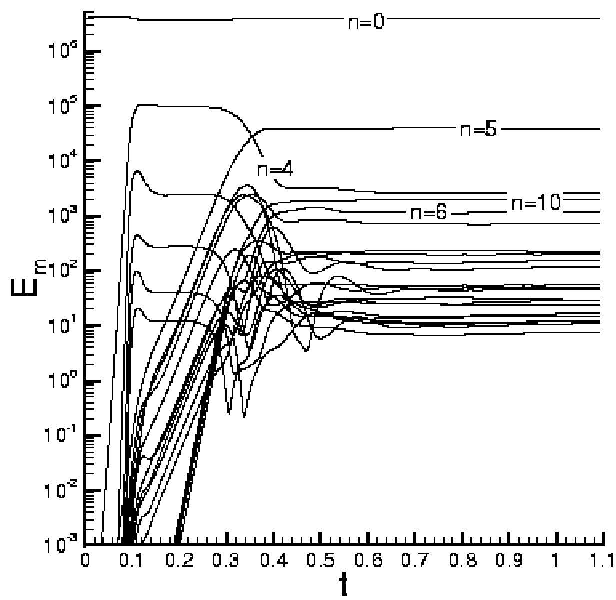
- The magnetic spectrum for the toroidal case shows that while the $m/n=1/n_p$ helicities have a large fraction of the energy, linear poloidal coupling (among the same n-values) is also quite significant.

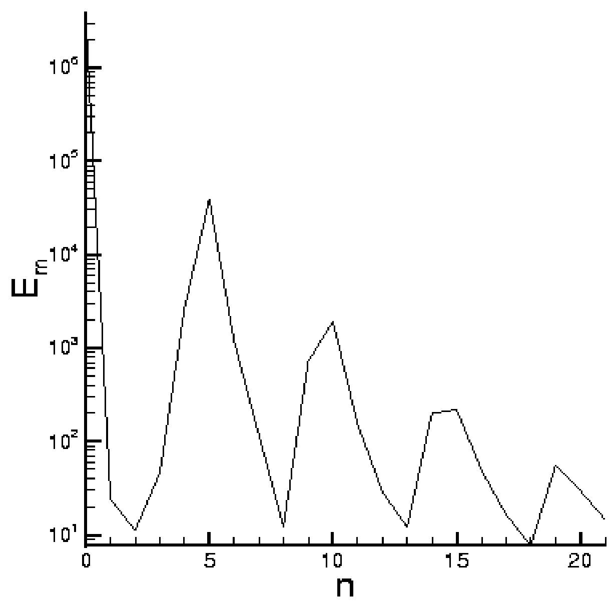


- When P_m is increased another order of magnitude, the toroidal simulation loses reversal. With m=0 fluctuations no longer resonant, a helical island chain forms in the interior.

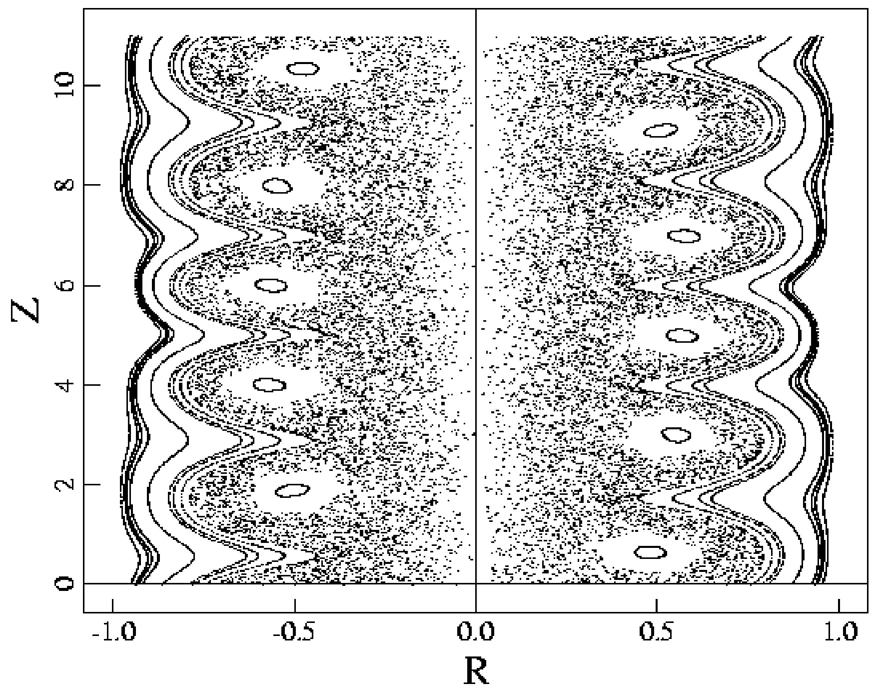


- In a periodic linear geometry simulation with P_m=10, Θ=1.65, and R/a=1.75, the final state is steady, but not single helicity.

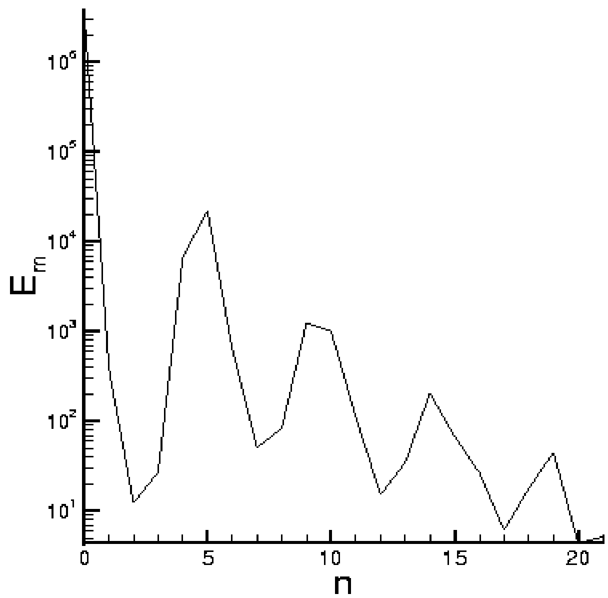


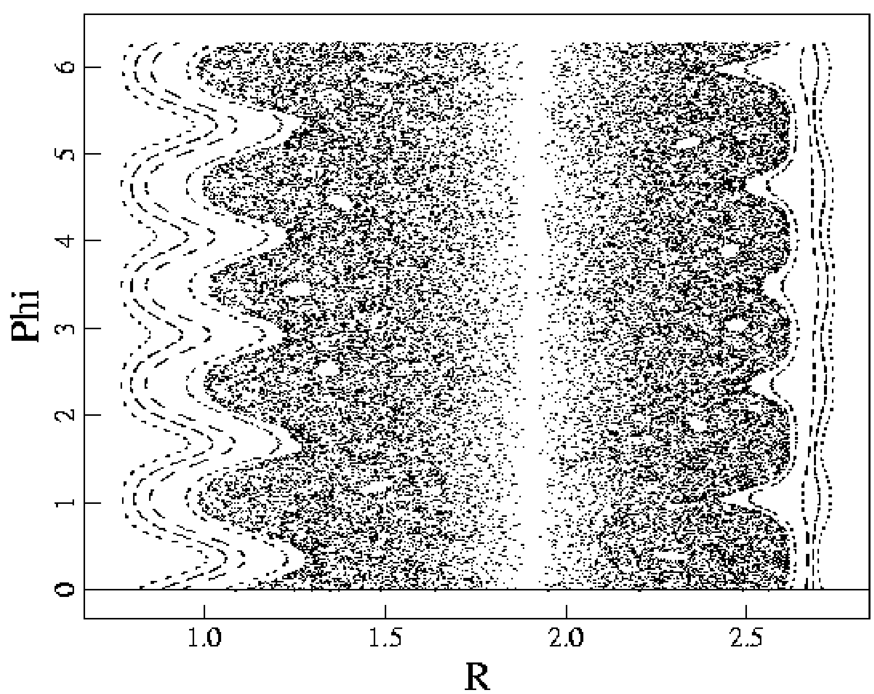


- These "quasi-single-helicity" states may show a coherent island structure by having a sufficiently *large* perturbation [Escande, *et al.*, "Chaos Healing by Separatrix Disappearance and Quasisingle Helicity States of the Reversed Field Pinch," PRL **85**, 3169 (2000)].



- The spectrum in toroidal geometry with P_m=10, Θ=1.6, and *R/a*=1.75 is similar, and a helical structure may be evident. However, *F*>0 and the lack of *m*=0 resonance is important.





Remaining Questions

1. What value of *R/a* is sufficiently large for the formation of laminar states with large island structures?

2. Can the strong poloidal coupling at low *R/a* be used to suppress fluctuations through sheared flow?

Conclusions

1. In typical multi-helicity RFP operation, toroidal geometry plays a minor role in comparison to nonlinear coupling.

2. In laminar conditions, toroidal geometry can make a qualitative difference. Conditions producing nonstochastic magnetic field in periodic linear geometry may have large regions of stochastic field in toroidal geometry.

3. The narrowing of the fluctuation spectrum as *R/a* is decreased does not indicate a transition to single helicity when toroidal geometry effects are considered.

4. The strong coupling at low *R/a* may be an opportunity for reducing fluctuation levels through shear flow. This remains to be explored.

This poster will be available through our web site, http://nimrodteam.org, shortly after the meeting.