# SIMULATING EXTREME ANISOTROPY WITHOUT MESH ALIGNMENT

**Carl Sovinec**
*Univ. of Wisconsin-Madison*

**Tom Gianakon**
*Los Alamos National Laboratory*

**MHD Working Group, Oct. 28 2001**
**APS 2001, Long Beach CA**

- Nonlinear simulations of high-temperature plasmas must be able to resolve thermal transport anisotropy associated with the magnetic field direction.
  - The saturation of pressure-driven modes is sensitive to changes in magnetic topology due to parallel thermal conduction.
  - The ratio of parallel to perpendicular thermal conductivities leads to one threshold mechanism for neoclassical tearing modes.
- When the magnetic field is aligned with the grid or when the angle between $\hat{\mathbf{b}}$ and the grid is uniform, the anisotropy is accurately represented by standard techniques.
- A nonlinearly evolving magnetic topology requires more sophisticated approaches:
  - There is curvature in the magnetic field.
  - The topology of islands and stochastic regions is three-dimensional.
  - Even 3D automated mesh refinement schemes would be severely challenged by these conditions; 3D refinement is possible but 3D alignment near a separatrix is not.
- The **NIMROD** code has addressed this challenge by using high-order finite-element basis functions, which represent curvature with or without alignment.
  - The increase in spatial convergence rates with basis function order has been verified.
  - Spatial convergence rates are retained with nonuniform meshing.

A quantitative measure of the numerical error can be determined by a simple test problem.

- The simplest thermal conduction problem is a 2D box with Dirichlet boundary conditions and a source. If the source drives the lowest eigenfunction only,

  $$S(x,y) = 2\pi^2 \cos(\pi x)\cos(\pi y)$$

  in the domain

  $$-0.5 \le x \le 0.5, -0.5 \le y \le 0.5 \ ,$$

  it produces the temperature distribution

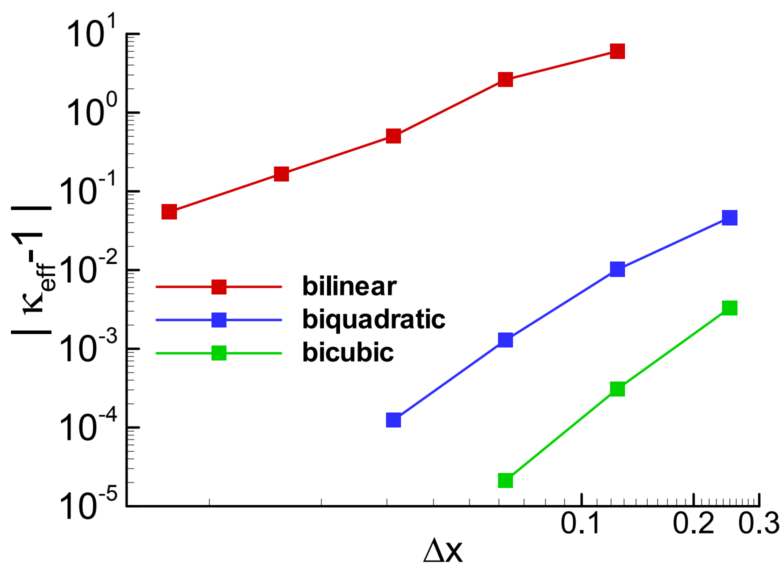
  $$T(x,y) = \kappa^{-1} \cos(\pi x)\cos(\pi y) \ .$$

- A numerical test with $\boldsymbol{B}$ everywhere tangent to this temperature distribution and a uniform rectilinear grid has $\hat{\mathbf{b}}$ severely misaligned with the grid. With $\kappa_\perp = 1$ and $\kappa_{||} >> 1$, the inverse of the resulting $T(0,0)$ is a good measure of the effective $\kappa_\perp$ of the numerical algorithm.

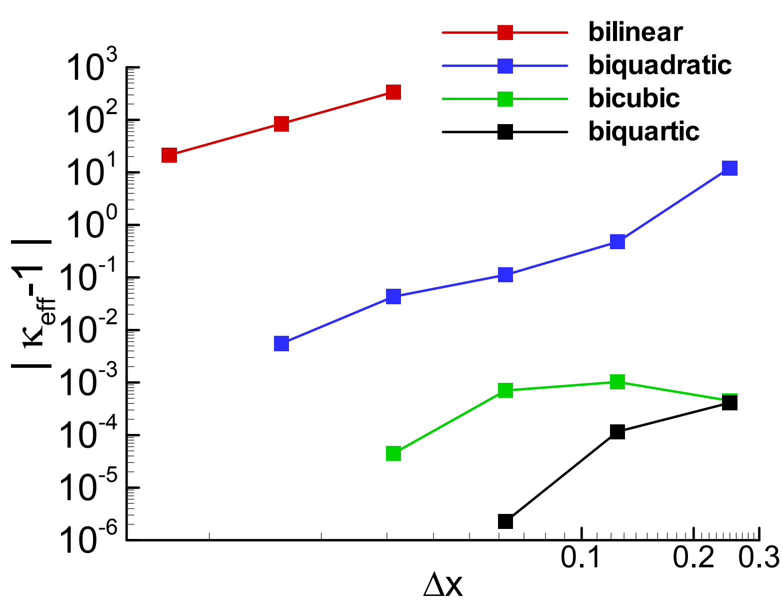
- The magnetic field is created by inducing a perpendicular current density distribution that is proportional to the heat source.

# Results from the Anisotropic Diffusion Test

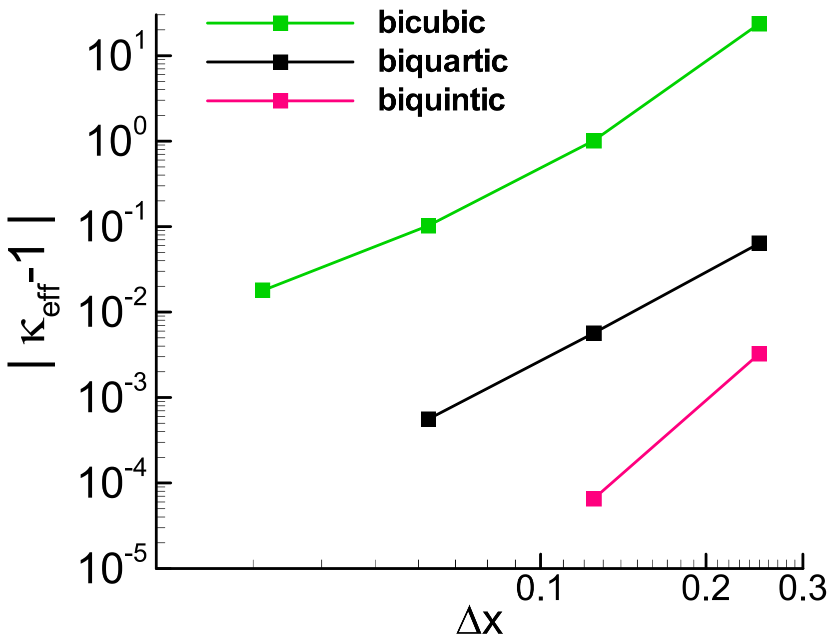
## Effective $\kappa_{perp}$-1 for $\kappa_{||} = 10^3$




## Effective $\kappa_{perp}$-1 for $\kappa_{||} = 10^6$



Effective $\kappa_{perp}$-1 for $\kappa_{||} = 10^9$



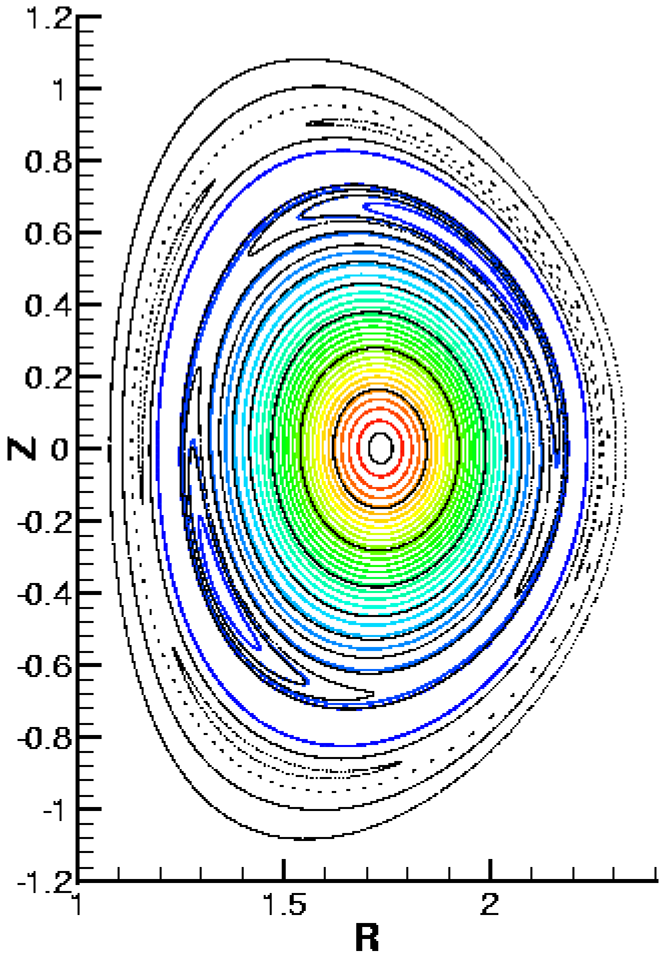

High-order basis functions enable resolution of extreme anisotropy in nonlinear simulations of electromagnetic fusion physics.

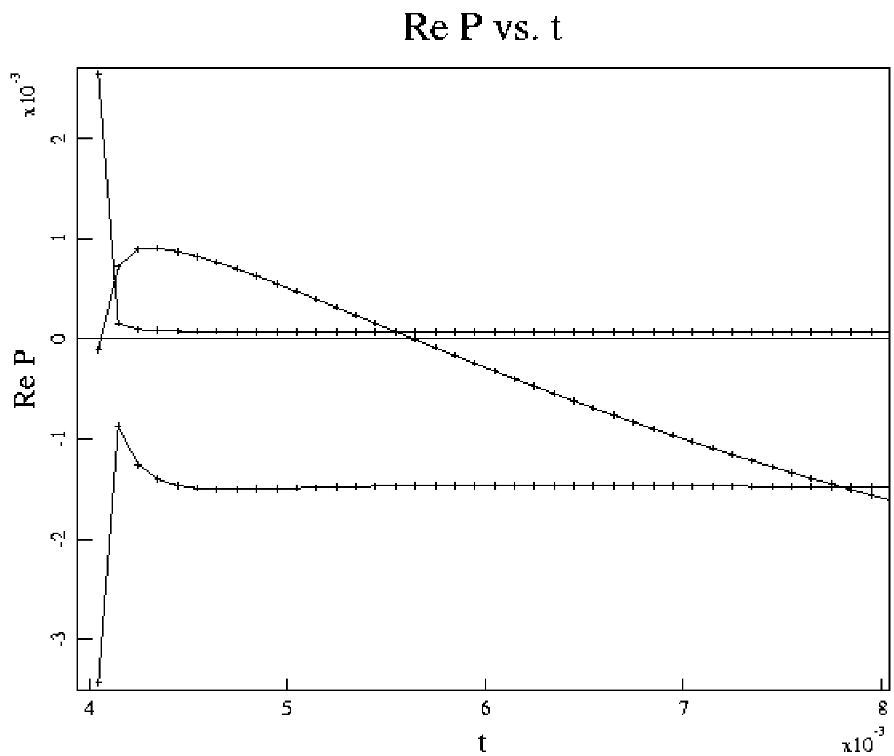
Finite elements also provide geometric flexibility.

## Anisotropic Diffusion Demonstration

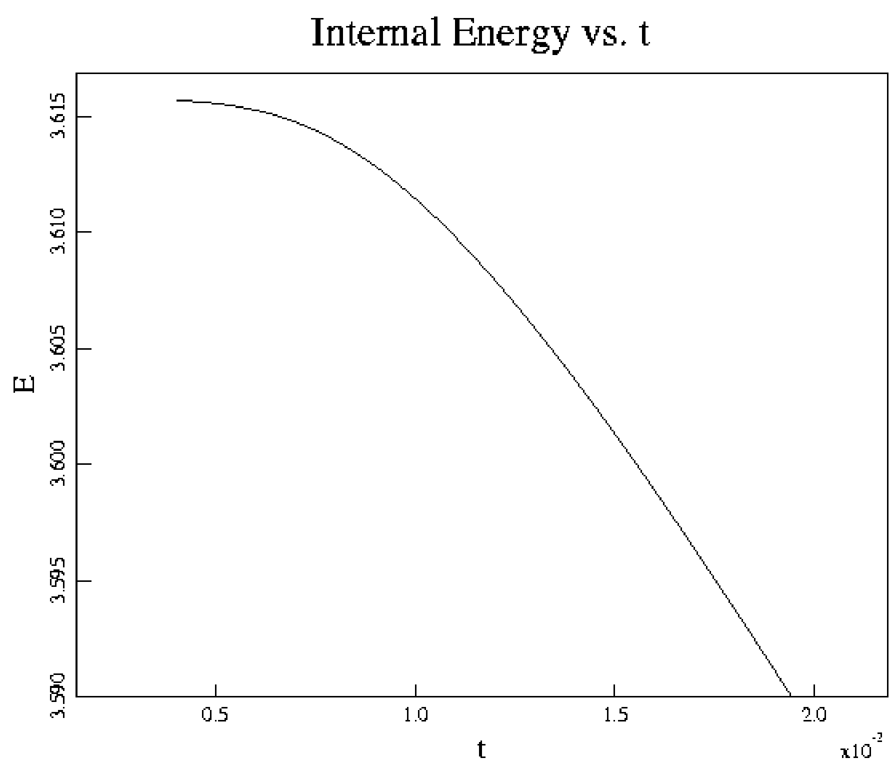
- Start from $S$=$10^4$, $P_m$=0.1 saturation of Test Problem 1b (DIII-D-like equilibrium) that had $\kappa_{iso}$=0.423 $m^2/s$=0.1 $\nu$.
- Freeze magnetic evolution and just run anisotropic thermal diffusion over the perpendicular time-scale (100 x resistive time-scale).
- $\kappa_\perp$=0.423, $\kappa_\parallel$=4.23x$10^8$, $\Delta$t=1x$10^{-4}$ s.
- Island appears in pressure contours immediately.

**Pressure contours (color) after 1.5 ms of anisotropic diffusion overlaid with Poincare surface of section.**





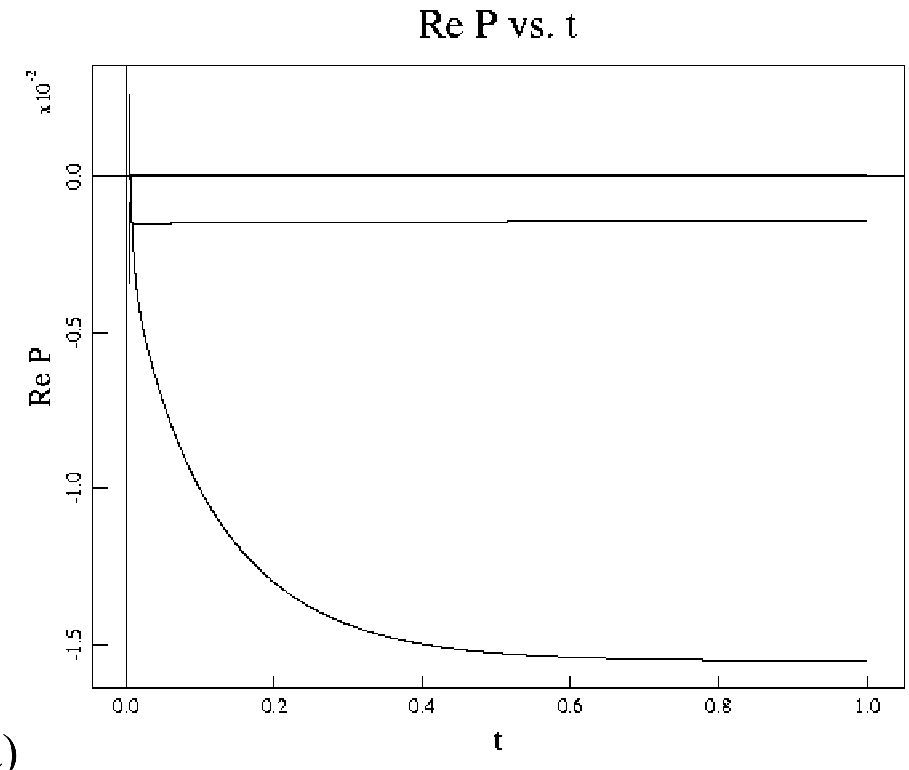

a)



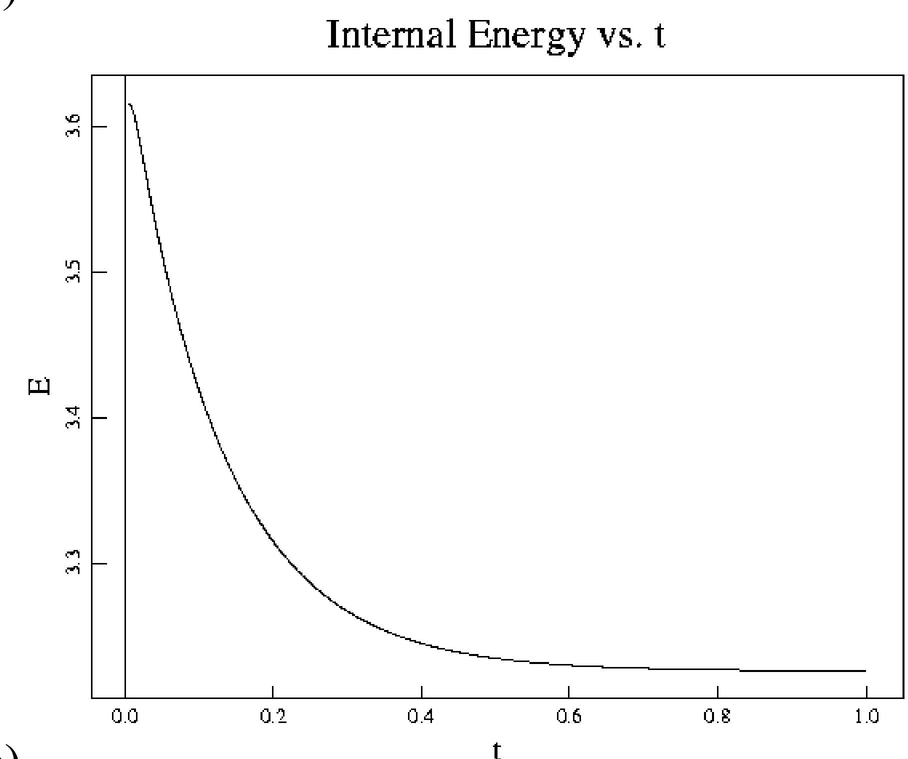

b)

a) "Probe" located at (R=1.312, Z=-0.248) and b) internal energy vs. time. Time-scale in a) is ms, and in b) it's 10 ms.

• Steady-state transport is regained after a perpendicular diffusion time.




a)




b)

a) "Probe" and b) internal energy. Time-scale is seconds.

- The $n$=0 pressure profile inside the island drops over the long time-scale, reflecting the loss of insulation over the magnetic island.