Kinetic shielding of magnetic islands in 3-D equilibria

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Abstract

Kinetic theory is used to describe pressure induced magnetic island formation in three-dimensional configurations. The kinetic responses describes currents that arise at rational surfaces that oppose the island producing magnetohydrodynamic (3-D) effects in general 3-D MHD equilibria. The kinetic response is largest at lowest collisionality suggesting that high-β stellarators are more resilient to retaining flux surface integrity at high-temperature than predictions from conventional MHD based theory would imply.

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I. Introduction

Understanding the properties of three-dimensional magnetohydrodynamic (MHD) equilibria is a crucial issue in stellarator physics. In particular, the breakup of magnetic surfaces due to pressure induced magnetic island formation and subsequent onset of magnetic stochasticity has been a topic of both analytic theory [1, 2, 3, 4, 5, 6] and computational studies for many years [7, 8, 9, 10]. The topic of the present work is the inclusion of kinetic closure modifications to pressure induced magnetic islands associated with 3-D equilibrium configurations. The kinetic response, which peaks at low collisionality, opposes the island producing mechanisms described by MHD theory and predicts smaller saturated magnetic island widths than that predicted by the associated fluid based calculations. This suggests that high-β stellarators are more resilient to retaining flux surface integrity at high-temperature than the associated predictions using conventional MHD theory would predict.

Since most present day computational tools available for studying 3-D equilibria with arbitrary magnetic topology rely on the MHD model [7, 8, 10], it is useful to review prior analytic treatments of magnetic surface breakup through the mechanism of magnetic island formation. Throughout the bulk of this work, the role of finite β effects will be emphasized. The analytic theory of pressure induced magnetic islands in 3-D equilibria using a resistive MHD model is well-developed [1, 2, 3, 5]. In general 3-D MHD equilibria with topologically toroidal magnetic surfaces, singular Pfirsch-Schlüter currents are generally present. For 3-D configurations, the scalar $1/B^2$ is a general function of all three space coordinates. If one assumes the existence of flux surfaces with a well defined $q(\psi)$ profile, $1/B^2$ can be written as the sum over Fourier modes using straight field line coordinates (ψ, θ, ζ), $1/B^2 = 1/B_{00}^2[1+\Sigma\delta_{mn}(\psi)e^{im\theta-in\zeta}]$. Solutions to the MHD quasi-neutrality equation $\nabla \cdot \mathbf{J} = \mathbf{B} \cdot \nabla(J_{||}/B) + \mathbf{B} \times \nabla p \cdot \nabla(1/B^2) = 0$ find that resonant components of the Fourier decomposition of $1/B^2$ (i. e., Fourier harmonics m, n such that $q = m/n$ surface lies within the plasma) produce parallel currents that diverge at the rational surface $(J_{||}/B)_{mn} \sim (q-m/n)^{-1}p'\delta_{mn}$. If one allows an island to form at the rational surface, the resonant Pfirsh-Schlüter currents are resolved. The result is a helically resonant parallel current localized to the island region that can provide a self-consistent source for the magnetic island producing magnetic field. Additionally, nonresonant components of the current can couple through the 3-D geometry of the equilibria to produce island producing magnetic fields. Using simple scaling arguments, one finds that saturated island width w normalized to the shear length $q_o' = dq/d\psi$ is given generically by an equation of the form

$$|q_o' w|^2 = \sum_{mn} C_{mn}\delta_{mn} \tag{1}$$

where the dimensionless coefficients C_{mn} scale with the plasma pressure and depend upon the detailed geometric properties of the equilibria. One might interpret MHD equilibrium β limit calculations in stellarators through a scenario of overlapping magnetic islands driven by pressure induced currents.

Vacuum islands and other pressure driven magnetic island mechanisms have been included in analytic island estimates. Of particular relevance to high temperature stellarators is the presence of bootstrap current effects (the analog of neoclassical tearing modes (NTM) in tokamaks [11]). With the correct choice of rotational transform gradient, neoclassical effects can be stabilizing to island formation [4, 12, 13]. However, the bootstrap current stabilization physics requires the local helical deformation of the pressure profile in the vicinity of the magnetic island. Hence, it is sensitive to the local transport properties around the island [14, 15, 16]. Resistive interchange [2, 3] and polarization current [17, 18, 19] effects also require self-consistent island induced modifications to the pressure profile. Conversely, Pfirsch-Schlüter currents (both resonant and non-resonant) and are insensitive to details in the anisotropic transport model used [20]. The "$1/x$" type resonance of the resonant component (where x denotes the distance from the rational surface) is a result of the global properties of the magnetic configuration that determine $1/B^2$ and not dependent upon subtle magnetic island physics [9]. In this sense, the Pfirsch-Schlüter effect is a robust mechanism for magnetic surface breakup in 3-D equilibria.

Non-MHD effects can also produce $1/x$ type resonances at rational surfaces. In a fluid theory formulation, these effects would enter through the viscous stress tensor $\nabla \cdot \vec{\vec{\pi}}$ that produces a perpendicular current $\mathbf{J}_\perp = \mathbf{B} \times \nabla \cdot \vec{\vec{\pi}}/B^2$. Through the quasineutraility equation, this provides another 'source' term for parallel current resonances. To calculate this particular closure moment we will develop a kinetic theory for low collisionality plasmas in Section 3.

The kinetic equation of interest is similar to that used to describe the response to 3-D fields used in stellarator neoclassical transport theory. For quasi-symmetric or tokamak applications, this response is also used to calculate neoclassical toroidal viscosity (NTV) forces [21, 22, 23, 24, 25]. In these applications, the goal is to calculate the 'second' order [26, 27] transport fluxes which subsequently affect the toroidal flow profile evolution. It is these transport effects that previous authors have used to deduce the effects of NTV on magnetic island physics [21, 23, 28]

However, in addition to this dissipative response, there is a 'first' order reactive response from the kinetic solution. This response describes currents that flow within the magnetic surfaces that, while not producing any net transport, can influence island physics through the parallel currents that flow in accordance with the quasineutrality condition. In general, the kinetic response will have both dissipative and reactive contributions. The relative role of these contributions will depend upon the collisionality regime of the plasma. Just as in stellarator neoclassical transport or NTV theory, formulae for the reactive response can be calculated for each asymptotic regime (e. g., $\nu - \sqrt{\nu}, 1/\nu$, superbanana-plateau, etc.). In this work, we will exclusively work in the ν regime of neoclassical transport. In this small collisionality limit, the reactive response is largest. We will leave calculations of the other asymptotic collisionality regimes to later work.

The kinetic response produces a stabilizing effect on magnetic island formation. Further, this stabilizing response is largest at smallest collisionality which suggest that high temperature stellarators are more resilient to retaining flux surfaces than MHD calculations suggest. We note that the kinetic theory developed is similar to work performed in Ref. [29] where the tokamak response to an applied 3-D magnetic field using a simplied Krook colision operator was calculated. What is different in our calculation is the specific application to island physics.

In the following section, we briefly review the MHD analytic island theory pertinent to isolated magnetic islands in 3-D equilibria. Initially, the identification of singular currents as general solutions to the quasineutrality equation is made when topologically toroidal magnetic surfaces are assumed. The allowance for a magnetic island resolves the singular current response. In section 3, the kinetic modification to the MHD calculation is carried out. In this section a modified quasineutrality condition is derived that accounts for the kinetic distortion. A summary of the results and implications for stellarators is given in Section 4.

II. MHD Island Physics

In this section, we briefly review aspects of isolated magnetic island physics in 3-D equilibrium using the MHD model. In the following subsection, an equilibrium magnetic field is assumed to exist that has topologically toroidal magnetic surfaces. In general finite-β 3-D magnetic fields, singular parallel currents are present. As shown in Section II.B., the singular currents are resolved by allowing magnetic islands to form.

A. Singular currents

In this sub-section, the existence of an equilibrium magnetic field with good magnetic surfaces are assumed. The equilibrium magnetic field is defined using straight field line coordinates (ψ, θ, ζ) by

$$\mathbf{B}_o = q(\psi)\nabla\psi \times \nabla\theta + \nabla\zeta \times \nabla\psi, \tag{2}$$

where $q(\psi)$ the safety factor and is a flux function. In the following, Boozer coordinates [30] will be employed. This also allows the equilibrium magnetic field to be written

$$\mathbf{B}_o = G(\psi)\nabla\zeta + I(\psi)\nabla\theta + h(\psi, \theta, \zeta)\nabla\psi, \tag{3}$$

where G and I are flux functions. Here, h is proportional to the pressure gradient and related to Pfirsch-Schlüter responses. The Jacobian is given by

$$\sqrt{g} = \frac{Gq + I}{B^2} \equiv \frac{\gamma(\psi)}{B^2}. \tag{4}$$

The Jacobian $\sqrt{g}$ or equivalently $1/B^2$ can be Fourier decomposed

$$\sqrt{g} = \frac{\gamma(\psi)}{B_{00}^2(\psi)}[1 + \sum_{mn} \delta_{mn} e^{i(m\theta - n\zeta - \phi_{mn})}], \tag{5}$$

where the phase factor ϕ_{mn} will be treated in general as time dependent so that the effects of rotating MHD modes are allowed in the formulation. In the following, the magnetic field strength is considered to be nearly symmetric so that $B \approx B(\psi, \theta - N\zeta)$ to leading order with weak 3-D magnetic fields. Using this assumption the Fourier decomposition can be rewritten as

$$\sqrt{g} = \frac{\gamma}{B_{00}^2}[1 + \epsilon \cos(\theta - N\zeta) + \sum_{mn} \delta_{mn} e^{i(m\theta - n\zeta - \phi_{mn})}] \tag{6}$$

where the dominant harmonic is explicitly identified with the coefficient $\epsilon = 2\delta_{1,N}$ and coordinates are choosen such that $\phi_{1,N} = 0$. In the following, we treat all $\delta_{mn} \neq \delta_{1,N}$ as small relative to ϵ. While this approximation is not necessary to complete the MHD calculation, this simplification will be useful in simplifying the kinetic analysis of Section 3. In practice, the resonant components of the Fourier sum will be of primary interest.

The current profile is written

$$\mathbf{J} = \lambda\mathbf{B} + \frac{\mathbf{B} \times \nabla p}{B^2}. \tag{7}$$

The quasineutrality condition is written $\nabla\cdot\mathbf{J} = \mathbf{B}\cdot\nabla\lambda + \nabla\cdot(\mathbf{B}\times\nabla p/B^2) = 0$. Using the equilibrium field described above produces the relation

$$\frac{1}{\sqrt{g}}(\frac{\partial}{\partial\theta} + q\frac{\partial}{\partial\zeta})\lambda = -\frac{1}{\sqrt{g}}(G\frac{\partial}{\partial\theta} - I\frac{\partial}{\partial\zeta})\frac{\sqrt{g}}{\gamma}\frac{dp}{d\psi}, \tag{8}$$

where the equilibrium condition $p = p(\psi)$ is used. The parallel current profile quantity λ can be written as a Fourier series

$$\lambda = \sum_{mn} \lambda_{mn} e^{im\theta - in\zeta}. \tag{9}$$

The quasineutrality equation for each Fourier harmonic is given by

$$(m - nq)\lambda_{mn} = -\frac{dp}{d\psi}\frac{mG + nI}{B_{00}^2}\delta_{mn}, \tag{10}$$

The general solution of this equation has two types of singular currents at the rational surface $q = m/n$.

$$\lambda_{mn} = -\frac{dp}{d\psi}\frac{mG + nI}{B_{00}^2}\frac{\delta_{mn}}{m - nq} + \Delta'_{mn} A_{mn}\delta(\psi - \psi_{mn}). \tag{11}$$

The first term is the resonant Pfirsch-Schlüter current due to the inhomogeneous source created by the 3-D property of the magnetic field spectrum and the second term is a homogeneous solution to the quasineutrality condition for the resonant harmonic of the parallel current. While in practice, the value of the resonant component δ_{mn} may in fact be quite small ($\delta_{mn} \sim 10^{-2} - 10^{-3}$), it nonetheless provides a mechanism for singular plasma currents in ideal MHD. As shown in the following section, these singularities are resolved by allowing for magnetic islands at the rational surface.

The second term in Eq. (11) proportional to the δ-function contribution is equivalent to the "exterior" region solution of low-β tearing mode analysis. This solution is asymptotically matched to an inner region island solution accounting for island resolved currents that flow in response to polarization currents, bootstrap effects, resistive interchange physics, etc. In all of the following analysis, these effects will be ignored for simplicity.

B. Island resolved currents

In order to resolve the singularity, the magnetic field is modified to allow for the formation of an isolated island chain at the $q = m_o/n_o$ surface.

Assuming the existence of this island, a current profile is constructed consistent with this magnetic field. The calculation is then made self-consistent by using Ampere's law to relate the helical current profile in the island region to the island producing magnetic perturbation.

Magnetic islands are allowed to form at the rational surface $q(\psi_o) = q_o = m_0/n_0$ due to the presence of a symmetry breaking magnetic field of the form $\sqrt{g}\mathbf{B}_1 \cdot \nabla\psi = n_o A \sin(m_o\theta - n_o\zeta - \phi_o)$. It is convenient to switch coordinate systems to

$$\alpha = \zeta - q_o\theta + \frac{\phi_o}{n_o}, \tag{12}$$

$$\chi = \frac{\theta - N\zeta}{1 - Nq_o}. \tag{13}$$

As such the total magnetic field given is given by the sum of magnetic island producing field and equilibrium field described in Eq. (2).

$$\mathbf{B} = \nabla\alpha \times \nabla\psi + \nabla\Psi^* \times \nabla\chi, \tag{14}$$

where the helical flux function Ψ^* is defined by

$$\Psi^* = \int (q - q_o)d\psi - A\cos(n_o\alpha) \approx q_o' \frac{x^2}{2} - A\cos(n_o\alpha). \tag{15}$$

Here $q_o' = dq/d\psi|_{\psi=\psi_o}$, $x = \psi - \psi_o$ and terms higher order in the island width are ignored. The island width in flux space is given by

$$w = 4\sqrt{|\frac{A}{q_o'}|}. \tag{16}$$

The magnetic island width is considered small relative to equilibrium scales (i. e., $|q_o' w| \ll 1$). Throughout this work, the small island approximation will be employed to simplify the analysis.

In the presence of the magnetic island, the magnetic field lines lie on surfaces of constant Ψ^*. An island rotational transform $\Omega(\Psi^*)$ can be introduced that describes the shear of the magnetic fields on helical surfaces relative to the magnetic island [31]. Assuming A is weakly dependent on x through the island region (i. e., the constant Ψ approximation of tearing mode analysis), the island rotational transform can be written

$$\Omega(\Psi^*) = \Omega(\Phi^*) = \frac{1}{\oint (d\alpha/2\pi)[1/\partial\Psi^*/\partial x]} = \pm\frac{\pi q_o' w}{4kK(k)}, \tag{17}$$

where $K(k)$ is the complete elliptic function of the first kind and

$$k^2 = \frac{2A}{A + |\Psi^*|}, \tag{18}$$

labels helical magnetic flux surfaces outside the magnetic island. An equivalent expression can be derived for surfaces inside the separatrix. Additionally, we can define an "x"−like variable Φ^* given by

$$\Phi^* = \pm\frac{wE(k)}{\pi k}, \tag{19}$$

where $E(k)$ is the elliptic integral of the second kind. This variable is defined to be consistent with

$$\frac{d\Psi^*}{d\Phi^*} = \Omega \tag{20}$$

The $+(-)$ signs in the definition of Ω and Φ^* correspond to $x > 0$ ($x < 0$). An angle-like variable α^* is introduced and defined by

$$\alpha^* = \frac{\pi}{n_o K(k)} F(\frac{n_o\alpha}{2}, k), \tag{21}$$

where F is the elliptic integral of the first kind. The quantites Φ^* and α^* are defined such that

$$\nabla\Phi^* \times \nabla\alpha^* = \nabla\psi \times \nabla\alpha. \tag{22}$$

$$\frac{\partial x}{\partial\Phi^*} \approx \frac{\Omega}{q_o' x(\Psi^*, \alpha)} = \frac{1}{x \oint \frac{d\alpha}{2\pi}\frac{1}{x}}, \tag{23}$$

$$\frac{\partial\alpha}{\partial\alpha^*} \approx \frac{q_o' x}{\Omega} = x \oint \frac{d\alpha}{2\pi}\frac{1}{x}. \tag{24}$$

In the integral expressions, the quantity $x = \pm\sqrt{(2/q_o')(\Psi^* + A\cos(n_o\alpha)}$ is expressed as a function of Ψ^* and α. Using these coordinates, the magnetic field with magnetic islands can be written

$$\mathbf{B} = \nabla\alpha^* \times \nabla\Phi^* + \Omega\nabla\Phi^* \times \nabla\chi. \tag{25}$$

The magnetic spectrum can also be written using the coordinates α^* and χ

$$\sqrt{g} = \frac{F}{B^2} = \frac{F}{B_{00}^2}[1+\epsilon\cos(1-Nq_o)\chi+\sum_{mn}\delta_{mn}e^{i(m-nq_o)\chi-in^*\alpha(\alpha^*)-i\phi_{mn}+i\phi_o n^*/n_o}], \tag{26}$$

where $n^* = (n - mN)/(1 - Nq_o)$. For $N = 0$ or for $m/n = q_o$, $n^* = n$.

The quasineutrality equation is now revisited. The major difference between the calculation of this section and that carried out in Section II.A is the existence of a magnetic island chain and the associated use of the magnetic field described in Eq. (25) with the coordinates Φ^*, χ, α^*. Using this form, the quantity $\mathbf{B} \cdot \nabla(J_{||}/B) = \mathbf{B} \cdot \nabla\lambda$ is written

$$\mathbf{B} \cdot \nabla\lambda = \frac{1}{\sqrt{g}}[\frac{\partial\lambda}{\partial\chi} + \Omega\frac{\partial\lambda}{\partial\alpha^*}]. \tag{27}$$

Using $p = p(\Phi^*)$, the quasineutrality condition becomes

$$\frac{\partial\lambda}{\partial\chi} + \Omega\frac{\partial\lambda}{\partial\alpha^*} - \frac{\partial}{\partial\alpha^*}\frac{\gamma p'}{B^2} + \frac{\partial}{\partial\chi}(\frac{G + NI}{1 - Nq_o}\frac{q_o' x}{\Omega}\frac{p'}{B^2}) = 0, \tag{28}$$

where $p' = dp/d\Phi^*$. The resonant and non-resonant portions of the parallel current profile can be separated using $\lambda = \bar{\lambda} + \tilde{\lambda}$ where

$$\bar{\lambda} = \oint \frac{d\chi}{2\pi}\lambda, \qquad \tilde{\lambda} = \lambda - \bar{\lambda}. \tag{29}$$

The non-resonant components $\tilde{\lambda}$ are largely unaffected by the presence of the island at $q = m_o/n_o$ and therefore these components are still described by the analysis of the previous section. The resonant component of the current then corresponds to $\bar{\lambda}$ which is now given by

$$\frac{\partial}{\partial\alpha^*}[\Omega\bar{\lambda} - \gamma p'\overline{B^{-2}}] = 0. \tag{30}$$

This has the general solution

$$\bar{\lambda} = \frac{\gamma p'}{\Omega}\overline{B^{-2}} + \Lambda(\Phi^*) = \frac{1}{\Omega}\frac{dp}{d\Phi^*}\frac{\gamma}{B_{00}^2}2\delta_{m_o,n_o}cos(n_o\alpha) + \Lambda(\Phi^*), \tag{31}$$

where $\Lambda(\Phi^*)$ is a yet undetermined function of the helical flux surface label Φ^* and the reality condition $\delta_{m_o,n_o} = \delta^*_{-m_o,-n_o}$ is used. In the zero island width limit, the island rotational transform asymptotes to $\Omega \rightarrow q_o' x$ and the familiar $1/x$ Pfirsch-Schlüter resonance is restored. However, with the presence of the magnetic island, the island rotational transform Ω is generally non-zero with only a weak logarithmic divergence at the island separatrix. Effects from resistive interchange physics [2, 3, 5] and polarization currents [17, 18, 19] are ignored in the above expression for simplicity.

The integration function $\Lambda(\Phi^*)$ is typically calculated in analytic island theory by using the constraints imposed by a non-ideal MHD Ohm's law.

Assuming a resistive MHD Ohm's law without the presence of bootstrap currents, inductive electric fields or RF currents, the parallel current obeys $\mathbf{E}\cdot\mathbf{B} = \eta\mathbf{J}\cdot\mathbf{B}$ with resistivity η. Λ can be calculated by averaging this equation over the helical magnetic surfaces of the island. This produces the condition

$$\oint \frac{d\chi}{2\pi} \oint \frac{d\alpha^*}{2\pi} \sqrt{g}\eta\mathbf{J}\cdot\mathbf{B} = 0, \tag{32}$$

which yields

$$\bar{\lambda} = \frac{1}{\Omega}\frac{dp}{d\Phi^*}\frac{2\gamma\delta_{m_o n_o}}{B_{00}^2}[\cos(n_o\alpha) - \oint \frac{d\alpha}{2\pi}\frac{\Omega}{q_o' x}\cos(n_o\alpha)]. \tag{33}$$

Recall, from Eq. (24), $\oint d\alpha^* \cos(n_o\alpha) = \oint d\alpha(\Omega/q_o' x)\cos(n_o\alpha)$.

Ampere's law $[\nabla\times(\nabla\times\mathbf{A}) = \mu_o\mathbf{J}]$ can be used to find a self-consistent magnetic island width due to the pressure induced currents. This results in the result given in Eq. (1).

In calculating this response, the pressure is assumed to equilibrate along the helical magnetic surfaces so that $p = p(\Phi^*)$ and pressure is constant inside the island separatrix. However, if transport processes perpendicular to the magnetic surfaces compete with transport processes along field lines, the pressure profile in the vicinity of the magnetic island will have a more complex structure [14, 15]. However, as pointed out in Ref. [20], this modification only only weakly affects the contribution to the island width from the resonant component of the Pfirsch-Schlüter current. The reason for this is that the source of helical nature of the current is due to the helically resonant component of the magnetic field spectrum.

III. Kinetic Theory

In this section, kinetic modifications to island physics are considered. The effect of 3-D magnetic fields is to produce net bounce averaged motion of trapped particles off of the flux surfaces. With a background Maxwellian plasma $f^0 = f_M(\Phi^*)$, the associated linearized drift kinetic equation for the kinetic distortion is of the form $\mathcal{L}(f^1) - C(f^1) = - < \mathbf{v}_d\cdot\nabla\Phi^* > \partial f_M/\partial\Phi^*$, where the bracket denotes a bounce average and is defined in Eq. (48). Solutions to this equation for f^1 are caused by the $\mathcal{O}(\delta)$ 3-D components of the magnetic field spectrum. In quasi-symmetric limits of stellarator neoclassical transport or equivalently NTV of near symmetric tokamaks, this f^1 solution is subsequently used to calculate non-ambipolar particle fluxes, $\vec{\Gamma}_s\cdot\nabla\Phi^* \sim \int d^3\mathbf{v}\,\mathbf{v}_d\cdot\nabla\Phi^* f_1$ that are second order in δ. What is calculated in the following is the $\mathcal{O}(\delta)$ plasma flows resulting from this kinetic distortion. While these flows do not produce any net radial transport, these flows produce currents that vary within flux surfaces and affect magnetic island physics through the parallel currents that respond to them due to quasineutrality.

Two classes of 3-D magnetic fields can produce net radial particle drifts. The first is the $\mathcal{O}(\delta)$ contributions to the magnetic field spectrum as described by Eq. (6). The second is due to the magnetic island's self-consistent deformation of the magnetic field spectrum as initially pointed out by Shaing [21]. The largest component of the magnetic spectrum is due to the symmetric components $B_{00}^2/B^2 = 1+\epsilon(\psi)\cos[(1-Nq_o)\chi]+\mathcal{O}(\delta)$. Noting that in the vicinity of the magnetic island $\psi \approx \psi_o + x = \psi_o \pm \sqrt{2(\Psi^* + A\cos(n_o\alpha)/q_o'}$, we have

$$\frac{B_{00}^2}{B^2} = 1 + \epsilon(\psi_o + x)\cos[(1 - Nq_o)\chi] + \mathcal{O}(\delta),$$

$$\approx 1 + \{\epsilon(\psi_o) + \frac{d\epsilon}{d\psi}x\}\cos[(1 - Nq_o)\chi] + \mathcal{O}(\delta). \quad (34)$$

The term proportional to x produces a 3-D component of order $wd\epsilon/d\psi$ that produces net radial particle drifts and provides a source for f^1.

The quasineutrality condition is again of the form $\mathbf{B}\cdot\nabla\lambda + \nabla\cdot\mathbf{J}_\perp = 0$ which can be derived from integrating the drift kinetic equation over velocity space and summing species. This yields

$$\mathbf{B}\cdot\nabla\lambda + \nabla\cdot\sum_s q_s\mathbf{v}_D f_s = 0, \quad (35)$$

where $\mathbf{v}_D$ is the sum of the $\mathbf{E}\times\mathbf{B}$, ∇B and curvature drifts. Using the coordinates convenient for describing magnetic islands, this equation becomes

$$\frac{\partial\lambda}{\partial\chi} + \Omega\frac{\partial\lambda}{\partial\alpha^*} + \frac{\partial}{\partial\Phi^*}\sum_s q_s\int d^3\mathbf{v}\sqrt{g}\mathbf{v}_D\cdot\nabla\Phi^* f_s$$

$$+\frac{\partial}{\partial\alpha^*}\sum_s q_s\int d^3\mathbf{v}\sqrt{g}\mathbf{v}_D\cdot\nabla\alpha^* f_s + \frac{\partial}{\partial\chi}\sum_s q_s\int d^3\mathbf{v}\sqrt{g}\mathbf{v}_D\cdot\nabla\chi f_s = 0. \quad (36)$$

Since we are primarily interested in the resonant component, we can average over χ and obtain

$$\Omega\frac{\partial\bar{\lambda}}{\partial\alpha^*} + \frac{\partial}{\partial\Phi^*}\sum_s q_s\oint\frac{d\chi}{2\pi}\int d^3\mathbf{v}\sqrt{g}\mathbf{v}_D\cdot\nabla\Phi^* f_s$$

$$+\frac{\partial}{\partial\alpha^*}\sum_s q_s\oint\frac{d\chi}{2\pi}\int d^3\mathbf{v}\sqrt{g}\mathbf{v}_D\cdot\nabla\alpha^* f_s = 0. \quad (37)$$

Using the magnetic field representation for the island equilibrium, this equation can be rewritten

$$\Omega\frac{\partial\bar{\lambda}}{\partial\alpha^*}+\frac{\partial}{\partial\Phi^*}\sum_s\oint\frac{d\chi}{2\pi}\int d^3\mathbf{v}m_s(\frac{v_{\|}^2}{2}+\frac{v_{\perp}^2}{4})(-\frac{\partial}{\partial\alpha^*}\frac{\gamma}{B^2}+\frac{G+NI}{1-Nq_o}\frac{q_o'x}{\Omega}\frac{\partial}{\partial\chi}\frac{1}{B^2})f_s$$

$$+\frac{\partial}{\partial\alpha^*}\sum_s\oint\frac{d\chi}{2\pi}\int d^3\mathbf{v}m_s(\frac{v_{\|}^2}{2}+\frac{v_{\perp}^2}{4})(\gamma\frac{\partial}{\partial\Phi^*}\frac{1}{B^2}-\frac{h\Omega}{q_o'x}\frac{\partial}{\partial\chi}\frac{1}{B^2})f_s=0. \quad (38)$$

If the distribution function is Maxwellian [$f_s = f_M(\Phi^*)$] the integrals can be evaluated. This expression simplifies noting the usual cancellation of the magnetization currents and leaves the following expression after making a small island width expansion

$$\frac{\partial}{\partial\alpha^*}[\Omega\bar{\lambda}-p'\gamma\overline{B^{-2}}]=0. \quad (39)$$

This equation is identical to the MHD solution given in Eq. (30).

In addition to the MHD current described above, there is an additional contribution due to a non-Maxwellian response. In order to make further analytic progress, we assume that the magnetic field to leading order is quasisymmetric. With this approximation, we order different contributions of the magnetic field spectrum Eq. (6) and its associated impact in describing $\mathbf{v}_d$. The leading order spectrum is simply given by $B_{00}^2/B^2 = 1+\epsilon[(1-Nq_o)\chi]$, while terms of order δ are higher order. Employing this approximation, the quasineutrality equation can be written

$$\mathbf{B}\cdot\nabla\lambda+\sum_s q_s\int d^3\mathbf{v}(\mathbf{v}_D^1\cdot\nabla f_M+\mathbf{v}_D^0\cdot\nabla f^1)=0, \quad (40)$$

where the middle term in the above expression corresponds to order δ terms associated with the magnetic drifts. This contribution was previously calculated and corresponds to the MHD solution, Eq. (30). What remains to be constructed is the solution for f^1 and its impact on the quasineutrality condition.

For the kinetic calculations it will be convenient to express the spectrum in Clebsch coordinates, with $\mathbf{B} = \nabla\beta\times\nabla\Phi^*$. With this, we define

$$\beta = \alpha^* - \Omega\chi, \quad (41)$$

and noting the smallness of Ω near the island region

$$\alpha(\alpha^*)\approx\alpha(\beta)+\frac{\partial\alpha}{\partial\alpha^*}\Omega\chi\approx\alpha(\beta)+q_o'x\chi, \quad (42)$$

which allows one to write

$$\frac{1}{B^2} = \frac{1}{B_{00}^2}[1+(\epsilon+x\epsilon')\cos(1-Nq_o)\chi+\sum_{mn}\delta_{mn}e^{i\Delta_{mn}\chi-in^*\alpha(\beta)-i\Delta\phi_{mn}}]. \quad (43)$$

where $\epsilon' = d\epsilon/\psi, \Delta_{mn} = m - nq_o - n^* q_o' x$ and $\Delta\phi_{mn} = \phi_{mn} - n^*\phi_o/n_o$.

The electrostatic potential in the island region is also required to describe guiding center drifts. To calculate the electric field required to construct the $\mathbf{E}\times\mathbf{B}$ drift we write $\mathbf{E} = -\partial\mathbf{A}/\partial t - \nabla\phi_E$ and use an ideal MHD Ohm's law to approximate the behavior

$$\mathbf{E}\cdot\mathbf{B} = -\mathbf{B}\cdot\frac{\partial\mathbf{A}}{\partial t} - \mathbf{B}\cdot\nabla\phi_E = 0. \quad (44)$$

Using the magnetic field representation $\mathbf{B} = \nabla\alpha\times\nabla\psi + \nabla\Psi^*\times\nabla\chi$ and $\mathbf{B} = \nabla\times\mathbf{A}$ we have from parallel Ohm's law

$$\frac{\partial\phi_E}{\partial\alpha} = -\frac{\partial}{\partial\alpha}[\frac{\dot{\phi}_0}{n_0}x(\Phi^*,\alpha)], \quad (45)$$

which can be integrated to yield [17, 18]

$$\phi_E = -\frac{\dot{\phi}_0}{n_0}x + h(\Phi^*). \quad (46)$$

In the island rest frame, the electrostatic potential is a helical flux function. At large x, the lab frame solution for the potential must asymptote to $\phi \rightarrow \phi_E' x$. Hence at large x, $h \rightarrow (\phi_E' + \dot{\phi}_0/n_o)x$.

Trapped particles produce the largest non-Maxwellian contribution to f^1. The bounce averaged kinetic equation for the trapped particles is given by

$$< \frac{\partial f^1}{\partial t} > + < \mathbf{v}_E\cdot\nabla\beta + \mathbf{v}_d^0\cdot\nabla\beta > \frac{\partial f^1}{\partial\beta} - < C(f^1) > = - < \mathbf{v}_d^1\cdot\nabla\Phi^* > \frac{\partial f_M}{\partial\Phi^*}. \quad (47)$$

The bounce averaging operator is defined by

$$< * > = \frac{\oint \frac{d\chi}{2\pi}\frac{\sqrt{g}B}{v_{||}} *}{\oint \frac{d\chi}{2\pi}\frac{\sqrt{g}B}{v_{||}}}, \quad (48)$$

Using $B \approx B_{00}[1 - \frac{\epsilon}{2}\cos(1-Nq_o)\chi]$, the bounce time for trapped particles can be written

$$\tau_b = \oint \frac{d\chi}{2\pi}\frac{\sqrt{g}B}{v_{||}} = \frac{\gamma}{B_{00}}\frac{1}{|1-Nq_o|}\sqrt{\frac{m_s}{2\mu B_{00}\epsilon}}\frac{2K(\kappa)}{\pi}. \quad (49)$$

where

$$\kappa^2 = \frac{E - \mu B_{00}(1-\epsilon/2)}{\mu B_{00}\epsilon} = \frac{E - \mu B_{min}}{\mu(B_{max} - B_{min})}. \tag{50}$$

The leading order drifts are given by

$$\mathbf{v}_E \cdot \nabla\beta = -\frac{dh}{d\Phi^*} + \frac{\Omega}{q_o' x}\frac{\dot{\phi}_0}{n_o} = \omega_E(\Phi^*) + \frac{\Omega}{q_o' x}\frac{\dot{\phi}_0}{n_o}, \tag{51}$$

$$\mathbf{v}_d^0 \cdot \nabla\Phi^* = \frac{v_{||}}{B}\frac{1}{\sqrt{g}}\frac{\partial}{\partial\chi}(\frac{v_{||}}{B}\frac{m_s}{q_s}\frac{q_o' x}{\Omega}\frac{G+NI}{1-Nq_o}). \tag{52}$$

Bounce averaging $\mathbf{v}_d^0 \cdot \nabla\beta$ we obtain

$$< \mathbf{v}_d^0 \cdot \nabla\beta >= \frac{\Omega}{q_o' x}\omega_d^0 \tag{53}$$

where

$$\omega_d^0 = \frac{1}{q_s}\frac{\mu B_{00}}{2}\frac{d\epsilon}{d\psi} < f_\mu \cos(1-Nq_o)\chi >, \tag{54}$$

and $f_\mu = (2E - \mu B)/(\mu B_{00})$ is a dimensionless quantity of order unity for trapped particles.

The general solution for f^1 involves finding solutions to the bounce averaged kinetic equation. Analytic solutions to Eq. (47) can be obtained in different asymptotic limits. As discussed earlier, the contribution of interest corresponds to a reactive response that enters into the quasineutrality condition. Therefore, for simplicity, we will pursue a solution assuming that the collisional effect is small to lowest order. This corresponds to the ν regime of neoclassical transport or NTV theory (valid when $\nu_{eff} \ll \omega_E, \omega_d^0$ where ν_{eff} is the effective collision frequency). In this limit, the collision operator can be neglected in Eq. (47) and the solution for f^1 can be found. Those contributions to f^1 driven by $\mathcal{O}(\delta)$ contributions to the magnetic spectrum are given by

$$\sum_{mn} f_{mn}^1 = \frac{df_M}{d\Phi^*}\sum_{mn}\delta_{mn}e^{-i\Delta\phi_{mn}}\frac{(mG+nI)}{n^*\gamma}\frac{\mu B_{00}}{2q_s}a_{mn}\int d\alpha \frac{q_o' x}{\Omega}\frac{-in^* e^{-in^*\alpha}}{\Delta\omega_{mn} + \omega_d^0 + \omega_E\frac{q_o' x}{\Omega}}, \tag{55}$$

where $\Delta\omega_{mn} = \dot{\phi}_0/n_0 - \dot{\phi}_{mn}/n^*$ and

$$a_{mn} =< e^{i\Delta_{mn}\chi} f_\mu \frac{B^2}{B_{00}^2} > . \tag{56}$$

In constructing this solution, we used the small island approximation, $\beta \approx \alpha^*$ and Eq. (24).

Of particular significance in the sum is the contribution to f^1 driven by the resonant component of $1/B^2 \sim \delta_{m_o n_o}$. For this particular component of the sum, the α integration can be easily performed in two asymptotic limits. For $\omega_E \gg \omega_d^0$,

$$f^1_{m_o n_o} \approx \frac{df_M}{d\Phi^*}\frac{\mu B_{00}}{q_s}\frac{a_{m_0 n_0}\delta_{m_0 n_0}}{\omega_E}\cos(n_o\alpha), \tag{57}$$

In the other limit $\omega_E \ll \omega_d^0$,

$$f^1_{m_o n_o} \approx \frac{df_M}{d\Phi^*}\frac{\mu B_{00}}{q_s}\frac{a_{m_0 n_0}\delta_{m_0 n_0}}{\omega_d^0}\frac{16|q_o'|(x^3 - \oint \frac{d\alpha}{2\pi}x^3)}{3w^2\Omega}. \tag{58}$$

The island deformation of the flux surfaces produces a bounce averaged radial drift given by

$$< \mathbf{v}_d \cdot \nabla\Psi^* >_w = \frac{A\epsilon'}{\Omega}\frac{\mu B_{00}}{2q_s} n_o \sin(n_o\alpha) a_w \tag{59}$$

where

$$a_w = < \frac{B^2}{B_{00}^2} f_\mu \cos[(1 - Nq_o)\chi] > . \tag{60}$$

The kinetic distortion associated with this is given by

$$f^1_w = \frac{df_M}{d\Phi^*}\frac{A\epsilon'}{\Omega}\frac{\mu B_{00}}{2q_s} a_w \int d\alpha \frac{-n_o \sin(n_o\alpha)}{\omega_d^0 + \omega_E \frac{q_o' x}{\Omega}}. \tag{61}$$

Note that the distinguishing feature of this solution is that it is driven by the coefficient $A\epsilon'/\Omega \approx w\epsilon'$. Hence, this solution requires the existence of the island to contribute to the localized helical currents.

Inserting the solution for f_1 into the quasineutrality equation produces an equation of the form

$$\frac{\partial}{\partial\alpha^*}(\Omega\bar{\lambda} - p'\gamma\overline{B^{-2}}) \approx -\frac{\partial}{\partial\alpha^*}\sum_s \oint \frac{d\chi}{2\pi}\int d^3\mathbf{v}\frac{\mu B_{00} f_\mu}{2}\gamma\frac{\Omega}{q_o' x}\frac{\partial B^{-2}}{\partial\psi} f^1_s. \tag{62}$$

If we approximate the dominant contribution to the trapped particle response as being due to $f^1 \approx f^1_{m_o n_o} + f^1_w$, the quasineutrality condition is given by

$$\frac{\partial}{\partial\alpha^*}(\Omega\bar{\lambda} - p'\frac{2\gamma}{B_{00}^2}\{\delta_{m_o n_o}[\cos(n_o\alpha) - \mathcal{R}_{m_o n_o} g(\alpha)] - w\epsilon' k_c \mathcal{R}_w g_w(\alpha)\}) = 0. \tag{63}$$

where

$$\mathcal{R}_{m_on_o} = \sum_s \oint \frac{d\chi}{2\pi} \int d^3\mathbf{v} \frac{\mu B_{00} f_\mu}{4q_s} \cos(1 - Nq_o)\chi \frac{d\epsilon}{d\psi} \frac{f'_m \mu B_{00}}{p'} \frac{a_{m_on_o}}{\omega_E + \omega_d^0}, \quad (64)$$

$$g(\alpha) = \frac{\Omega}{q'_o x(\alpha)} \int^\alpha d\alpha' \frac{q'_o x(\alpha')}{\Omega} \frac{-n_o \sin(n_o\alpha)[\omega_E + \omega_d^0]}{\omega_d^0 + \omega_E \frac{q'_o x(\alpha')}{\Omega}}. \quad (65)$$

$$g_w(\alpha) = \frac{\Omega}{q'_o x} \int^\alpha d\alpha' \frac{-n_o d\alpha' \sin(n_o\alpha')[\omega_E + \omega_d^0]}{\omega_d^0 + \omega_E \frac{q'_o x}{\Omega}}, \quad (66)$$

$k_c = \pm kK(k)/4\pi$ and $\mathcal{R}_w$ is identical to $\mathcal{R}_{m_on_o}$ with the exception of a_w replacing $a_{m_on_o}$ in the integrand. Recall, in calculating $\mathcal{R}_w$, the integration in velocity space corresponds to the trapped particle contribution, thus $\int d^3\mathbf{v} = (8\pi^2/m_s^3)^{1/2} \int_0^\infty dE \int_{E/B_{max}}^{E/B} d\mu B(E - \mu B)^{-1/2}$, where $\mu = E/B_{max}$ denotes the trapped/passing boundary. Crudely, the kinetic correction factor scales as

$$\mathcal{R}_{m_on_o} \sim \mathcal{R}_w \sim \sum_s \sqrt{\epsilon} \frac{v_{ths}^2}{\Omega_{cs}} \frac{q}{rR(\omega_E + \omega_d^0)} \quad (67)$$

in the cylindrical flux surface limit where v_{ths} and Ω_{cs} are the thermal velocity and cyclotron frequency for species s and the trapped particle fraction scales with the square root of the inverse aspect ratio $\sqrt{\epsilon}$. For $\omega_E \sim \omega_d^0$, $\mathcal{R}_{m_on_o} \sim \mathcal{R}_w \sim \sqrt{\epsilon}$.

What Eq. (63) indicates is that the kinetic correction has a weak direct stabilizing effect on island formation through a modification of the resonant Pfirsch-Schlüter drive. Effectively, the resonant component of the magnetic spectrum becomes modified by the shielding factor $1 - \mathcal{R}_{m_on_o}$. Since $\mathcal{R}_{m_on_o}$ is typically less than unity, this provides a small correction. Additionally, there are components of the resonant kinetic current driven by nonresonant components of the magnetic field spectrum. There will be corresponding contributions to the kinetic response that are characterized by an equivilant kinetic correction factor $\mathcal{R}_{mn}$ that will entail integrals over the bounce average contribution a_{mn} described by Eq. (56). By similar arguments, these corrections are minor.

The most important kinetic contribution is due to the magnetic island producing island currents proportional to $\mathcal{R}_w$ in Eq. (63). Using this contribution as the dominant kinetic correction, a self-consistent island width accounting for kinetic modifications can be derived that is given by the equation

$$|q'_o w|^2 = \sum_{mn} C_{mn}\delta_{mn} - C_{m_on_o} w\epsilon' \mathcal{R}_w. \quad (68)$$

where the first term corresponds to the MHD prediction and the kinetic response is given by the last term that scales with the island width. The quantity $C_{m_o n_o}$ is a dimensionless quantity of order the poloidal β. In the limit $\mathcal{R}_w \to 0$, Eq. (1) is recovered. With small kinetic shielding, the saturated island width scales as $\delta_{mn}^{1/2}$. At moderate values of $\mathcal{R}_w$, the island width asymptotes to w_k where

$$w_k = \frac{\sum_{mn} C_{mn}\delta_{mn}}{C_{m_o n_o}\epsilon' \mathcal{R}_w}. \tag{69}$$

Since w_k scales with $\delta_{mn} \ll 1$, the kinetic effects substantially reduce the predicted island width. A plot of the island width as a function of $\mathcal{R}_w$ as given by the solution to Eq. (68) is given in Figure 1.

One can include the effects of bootstrap currents as well. Allowing for neoclassical corrections to steady state Ohm's law, the procedure described at the end of Section II can be repeated with the following result for the parallel current

$$\bar{\lambda} = \frac{dp}{d\Phi^*}\frac{2\gamma}{B_{00}^2}[\frac{\delta_{m_o n_o} G_0(\alpha)}{\Omega} - \frac{\epsilon'}{q_o'} k_w \mathcal{R}_w G_1(\alpha)] + \Lambda_{BC}(\Phi^*), \tag{70}$$

where $k_w = k^2 K(k)^2/\pi^2$

$$G_0(\alpha) = cos(n_o\alpha) - \oint \frac{d\alpha^*}{2\pi}\cos(n_o\alpha) - \mathcal{R}(g - \oint \frac{d\alpha^*}{2\pi} g) \tag{71}$$

$$G_1(\alpha) = g_w - \oint \frac{d\alpha^*}{2\pi} g_w \tag{72}$$

and the bootstrap current contribution in the long collision length limit is given by [4]

$$\Lambda_{BC} = -\frac{\sqrt{\epsilon}k_b}{1 - Nq_o}\frac{G + NI}{B_{00}^2}\oint \frac{d\alpha^*}{2\pi}\frac{\partial p}{\partial x} \tag{73}$$

with k_b a positive constant of order unity. For the quasisymmetric configuration of interest in this calculation, only the symmetric part of the magnetic field spectrum is used to calculate the island bootstrap. The reason that this current can contribute to island physics is due to the self-consistent deformation of the pressure profile in the island region. For large islands, $p = p(\Phi^*)$ and the pressure gradient is flat inside the island separatrix. In this case, bootstrap currents are not present inside the island, but are present ouside the separatrix. Due to the helical shape of the magnetic surfaces, the boostrap current current profile naturally produces a helical component resonant with the island. However, for island widths comparable or less than a critical width described by anisotropic transport, $w \sim w_D$ [14, 15, 16], the pressure profile does not equilibrate over the island flux surfaces and the helically resonant component of the pressure gradient and the corresponding bootstrap current is weakened. For transport processes assuming a Fick's law form $w_D \sim (\chi_\perp/\chi_{||})^{1/4}$ where $\chi_\perp$ and $\chi_{||}$ are the perpendicular and parallel diffusivities. This effect can be crudely accounted for through the parametrization $\oint \partial p/\partial x d\alpha^*/d\pi \approx p' w^2/(w^2 + w_D^2)$.

The second and third terms in Eq. (70) share the same radial parity. Whereas the second term is stabilizing to island formation under the usual conditions ($p'\epsilon' < 0$), the stability properties of the bootstrap effects depends upon the direction of the bootstrap current and sign of the magnetic shear. For the quasisymmetric configuration considered here, $p'q'(1 - Nq_o) < 0(>0)$ corresponds to destabilizing (stabilizing) contributions. Note that when this factor is negative (positive), the two currents oppose each other (add) at the island X-point which indicates the direction of stabilization. The ratio of the second and third terms in Eq. (70) is given crudely by the factor

$$-\frac{\epsilon'}{q_o'}\frac{\mathcal{R}_w}{\sqrt{\epsilon}}(1 - Nq_o)(1 + \frac{w_D^2}{w^2}). \tag{74}$$

For large islands ($w > w_D$) in tokamaks where $\epsilon'/q_o' \sim \epsilon$, this factor is small indicting the prominence of bootstrap current physics at high temperature. However, for small islands and/or small shear stellarators, this factor can be large indicating that the kinetic response can dominate bootstrap current effects.

Including the bootstrap current effect in the saturated island width equation yields

$$w^2 = \frac{\sum_{mn} C_{mn}\delta_{mn}}{q_o'^2} - \frac{C_{m_o n_o} w\epsilon' \mathcal{R}_w}{q_o'^2}. + D_{nc} w \frac{B_\theta R r}{m_o}\frac{w^2}{w^2 + w_D^2}, \tag{75}$$

where $D_{nc} \approx -\sqrt{\epsilon} k_1 \mu_o p'/[q_o' B_\theta^2 (1 - Nq_o)]$. In the absence of Pfirsch-Schlüter effects, saturated islands scale with the boostrap current contribution $w \sim D_{nc}$, if $D_{nc} > 0$. However, if $D_{nc} < 0$, large islands are opposed by bootstrap current effects. While the bootstrap current stabilization process is limited to islands larger than w_D, the kinetic response is operative at island widths below w_D.

IV. Summary

Kinetic theory is employed to calculate corrections to analytic predictions of saturated magnetic islands due to pressure gradients in 3-D magnetic configurations. The theory calculates the dominant trapped particle response to 3-D field induced net bounce averaged radial drifts. The associated kinetic response describes plasma currents that flow within magnetic surfaces. In general, these currents have a component that resonates with the helical angle of the magnetic island and affect saturated island sizes through the parallel currents generated to satisfy quasineutrality.

As indicated in Eq. (63), the kinetic correction has a small direct stabilizing effect on magnetic island formation through the presence of a shielding factor on the helically resonant Pfirsch-Schlüter current. This correction is measured by the quantity $\mathcal{R}_{m_o n_o}$ given by Eq. (64). More importantly, the magnetic island itself induces additional kinetic contributions through the self-consistent deformation of the flux surfaces. These deformations introduce an intrinsically 3-D component of the magnetic field strength that produces net bounce averaged radial drifts of trapped particles that are proportional to the island width. This effect produces additional currents in the island region that provide a stabilizing contribution to island formation. As shown in Eq. (68) and Figure 1, for moderate values of the kinetic shielding factor $\mathcal{R}_w$, the kinetic corrections provide a substantial stabilizing contribution to magnetic island formation at high β.

In order to calculate the kinetic response, a bounced averaged drift kinetic equation as given in Eq. (47) requires solution. In order to make analytic progress, a low collisionality approximation was used that corresponds to the associated ν regime of neoclassical stellarator transport. At larger values of collisionality, the kinetic response is reduced. Detailed calculations for different asymptotic collisionality regimes is left for future work.

The implication of this work is that at high temperature, kinetic corrections to magnetic island physics are significant in 3-D configurations. Tools that employ the MHD model to describe breakup of magnetic surfaces in 3-D equilibria may be unduly pessimistic. High temperature stellarators are more resilient to flux surface integrity than theoretical predictions using conventional MHD models would imply. In order to calculate the kinetic responses, closure calculations would need to be coupled to extended MHD numerical tools. In particular, to account for the effects discussed here, drift kinetic theory would be required to calculate the pressure anisotropy and the associated contributions to the fluid momentum balance through the term $\nabla \cdot \vec{\vec{\pi}}_s$.

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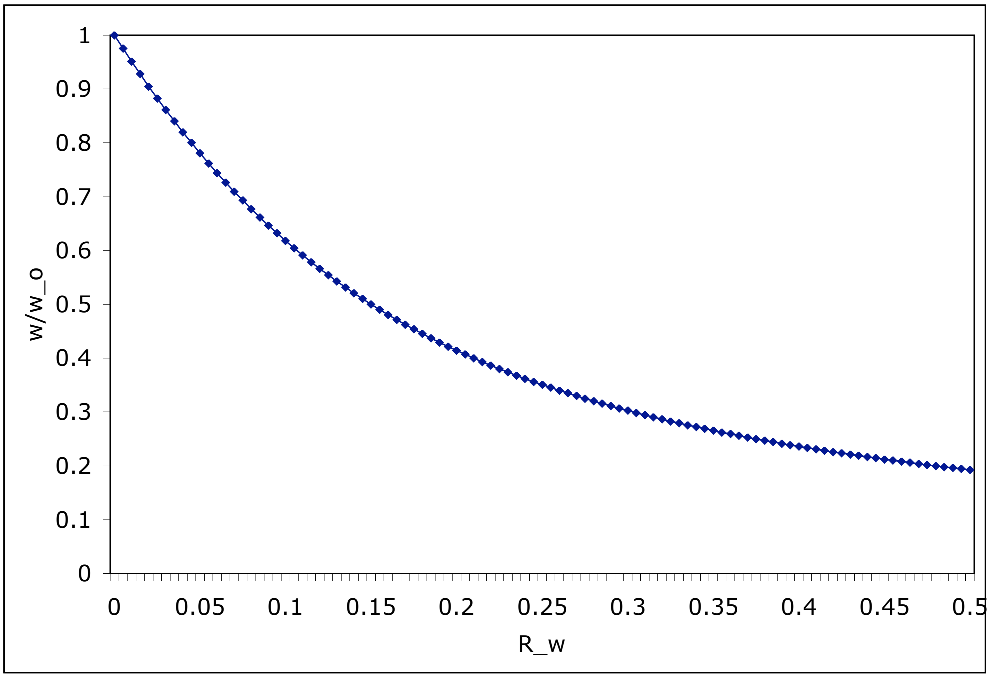

Figure 1. The value of the saturated island normalized to the MHD prediction $w/w_o = w|q_o'|/\sum_{mn} C_{mn}\delta_{mn}$ is plotted as a function of the kinetic correction factor $\mathcal{R}_w$. The plot is obtained as a solution to Eq. (68) with $\sum_{mn} C_{mn}\delta_{mn}q_o'^2/(C^2_{m_o n_o}\epsilon'^2) = 10^{-2}$. At moderate values of $\mathcal{R}_w$, the island width asymptotes to $w \to w_k$ where w_k is given by Eq. (69). For the parameters of this plot, $w_k/w_o = 0.1$.