A closure scheme for modeling RF modifications to the fluid equations

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Abstract

A procedure to include the effects of externally applied radio frequency (RF) sources in a comprehensive fluid model is outlined. The fluid equations are derived from moments of a kinetic equation that includes the effects of an RF source. In general, this source produces additional terms in each of the fluid equations. A complete derivation requires the specification of the closure moments; calculations for the stress tensors and heat fluxes that are altered by the presence of the RF are required. By treating the RF induced modification as producing a small distortion away from the background Maxwellian distribution function, a problem similar to the classic Spitzer problem can be formulated. Using a Chapman-Enskog-like procedure, a kinetic equation for the kinetic distortion can be derived that includes the RF-induced contributions to the fluid equations as sources that can be solved in a number of limiting cases.

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I. INTRODUCTION

A hallmark of present day tokamak operation is the use of control tools to improve plasma performance. This is particularly evident in the field of magnetohyrodynamic (MHD) mode control where various techniques using external magnetic fields, profile control, etc. have been developed to avoid and/or combat deleterious instabilities. An important example of crucial relevance to ITER [1] is the use of localized current drive [principally electron cyclotron current drive (ECCD)] to affect neoclassical tearing modes [2, 3] in high performance tokamaks. While simple theoretical models for this interaction exist to describe the phenomenology, quantitative predictions require the self-consistent descriptions of both the RF and MHD physics. In this work, we present a scheme to include the effects of externally applied RF sources in a comprehensive fluid model. The goal of this work is to provide proper theoretical foundation for the interaction of RF-wave physics with extended MHD models [4]. This work is largely motivated by the growing need to model the interaction of RF and MHD processes that require the self-consistent coupling of different classes of numerical tools [5, 6].

In an effort to combat the detrimental effect of neoclassical tearing modes in high performance tokamaks, a campaign to use localized current drive has proven effective on a number of major tokamak experiments [7, 8, 9, 10, 11]. Prior theoretical efforts to model this physics have relied on simplified theoretical models [12, 13, 14] or simplified numerical simulation techniques [15, 16]. These treatments typically employ an ad-hoc RF current drive source in Ohm's law. In order to describe the physics of localized current drive stabilization, past theoretical models consider an Ohm's law of the form

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta_{neo}(\mathbf{J} - \mathbf{J}_{bootstrap}) - \frac{\mathbf{F}_{rf}}{ne}, \tag{1}$$

where η_{neo} and $\mathbf{J}_{bootstrap}$ describes the neoclassical prediction for plasma resistivity and bootstrap currents, respectively. The effect of the RF source ($\mathbf{F}_{rf}$) enters as an additional force on the electron fluid. The essential physics of the RF stabilization for slowly growing magnetic islands can be demonstrated using a modified nonlinear island Rutherford theory that accounts for the additional ad-hoc force [12, 13]. These calculations produce an island evolution equation of the form [2, 3]

$$k_0 \frac{dw}{dt} = \frac{\eta}{\mu_o}[\Delta' + \frac{D_{nc}}{w} + D_{thresh}(w) - D_{rf}(w)], \tag{2}$$

where the island of interest has width w and can be driven as an unstable tearing mode ($\Delta' > 0$) or by neoclassical tearing mode effects [as described by D_{nc}]. The precise form for the threshold physics [$D_{thresh}(w)$] depends upon the details of the model and is not of central relevance to the topic of this work. The effect of the RF source modifies the island evolution in two ways. The current source can ultimately affect the global current profile and hence modify the asymptotic matching parameter Δ' [14]. If the RF source is sufficiently localized to the island region, the rapid flow of electrons along field lines produce a helically localized source in the vicinity of the island that can stabilize (or destabilize) the magnetic island. The details of this contribution D_{rf} depend upon the degree of spatial localization and phasing of the RF force relative to the island.

A first principles theoretical description of all of the physical processes involved in the interaction of RF and MHD physics is challenging owing to the enormous range of temporal and spatial scales. The role of the present work is provide a foundation that allows one to theoretically describe this interaction based on a fluid model formulation. The ability to come up with such a scheme is made possible by exploiting the fundamental time scale separation present in this problem. Electron cyclotron heating and current drive do produce modifications to the electron distribution function. However, these deviations are not so large as to produce massive deviations of the distribution function away from Maxwellian; as such, a theoretical formulation based on a fluid approach is still viable. In the following, the fluid equations are derived from moments of a kinetic equation that includes the effects of an RF source.

A kinetic equation relevant to the derivation of the fluid equations can be derived by averaging over the fast timescales of the RF. This produces a kinetic equation of the form $df/dt = C(f) + Q(f)$, where $C(f)$ and $Q(f)$ represent the collision operator and effects of the RF fields. One can take moments of this equation and construct the usual fluid equations. These equations are augmented by additional terms due to the RF source. A complete derivation requires the specification of the closure moments; calculations for the stress tensors and heat fluxes including RF effects are needed.

In the following section, we use the kinetic equation to derive the fluid equations. If we treat the RF term as producing a small distortion of the distribution function from Maxwellian, a Chapman-Enskog-like approach can be used. In Section III, this approach is employed to construct a kinetic equation to be used to calculate closure moments in the presence of RF sources. In Section IV, the kinetic equation is solved in a simple analytic limit; a problem similar to the classic Spitzer problem can be formulated. In Section V, a discussion of the results is given.

II. KINETIC EQUATION

In order to employ a useful kinetic equation for the derivation of fluid moment equations, we take advantage of the timescale separation between MHD-like and RF-like processes. Namely, a multiple timescale procedure is used where RF processes occur on a fast $\tilde{t}$ scale while MHD-like processes occur on a slower t timescale. An averaging procedure is defined over the fast RF timescale and is given by

$$\langle f \rangle = \oint \frac{d\tilde{t}}{\tau_{RF}} f, \tag{3}$$

where τ_{RF} is the periodicity time of the RF fields and $f = \langle f \rangle + \tilde{f}$. The kinetic equation for distribution function f_s can be written

$$\frac{\partial f_s}{\partial t} + \mathbf{v} \cdot \nabla f_s + \mathbf{a}_s \cdot \frac{\partial f_s}{\partial \mathbf{v}} = C(f_s), \tag{4}$$

where $\mathbf{a_s} = (q_s/m_s)(\mathbf{E} + \mathbf{v} \times \mathbf{B})$. The electromagnetic fields can be written $\mathbf{a}_s = \langle \mathbf{a}_s \rangle + \tilde{\mathbf{a}}_s$ with RF induced fields given by $\tilde{\mathbf{a}}_s$. Averaging the kinetic equation over the fast RF-timescale produces the evolution equation for the averaged distribution function

$$\frac{d\langle f_s \rangle}{dt} - \langle C(f_s) \rangle = -\frac{\partial}{\partial \mathbf{v}} \cdot \langle \tilde{\mathbf{a}}_s \tilde{f}_s \rangle, \tag{5}$$

where the effect of the RF-fields on $\langle f_s \rangle$ is given by the term on the right and

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla + \langle \mathbf{a_s} \rangle \cdot \frac{\partial}{\partial \mathbf{v}}. \tag{6}$$

Subtracting the averaged equation from the kinetic equation produces an evolution equation for $\tilde{f}_s$ of the form

$$\frac{d\tilde{f}_s}{dt} - C(f_s) + \langle C(f_s) \rangle = -\tilde{\mathbf{a}}_s \cdot \frac{\partial \langle f_s \rangle}{\partial \mathbf{v}} + \frac{\partial}{\partial \mathbf{v}} \cdot (\tilde{\mathbf{a}}_s \tilde{f}_s - \langle \tilde{\mathbf{a}}_s \tilde{f}_s \rangle). \tag{7}$$

Assuming $\tilde{f}_s \ll f_s$, the collision operator becomes a linear operator on $\tilde{f}_s$ and the nonlinear terms on the right can be ignored. The resulting linear equation for $\tilde{f}_s$ has a formal solution of the form $\tilde{f}_s = -\mathcal{L}_s^{-1} \tilde{\mathbf{a}}_s \cdot \partial \langle f_s \rangle / \partial \mathbf{v}$ where $\mathcal{L}_s$ is the linear operator produced from the above kinetic equation and $\mathcal{L}_s^{-1}$ is its inverse.

Using this solution for $\tilde{f}_s$ we arrive at a kinetic equation for $\langle f_s \rangle$ that can used as the starting point for the moment equation calculation.

$$\frac{d\langle f_s \rangle}{dt} = \langle C(f_s) \rangle + Q(\langle f_s \rangle), \tag{8}$$

where $Q(f_s)$ is the conventional quasi-linear diffusion operator with diffusion tensor $\mathbf{D}_s$ [19]

$$Q(f_s) = \frac{\partial}{\partial \mathbf{v}} \cdot \mathbf{J}_s^{v/rf} = \frac{\partial}{\partial \mathbf{v}} \cdot \mathbf{D}_s \cdot \frac{\partial f_s}{\partial \mathbf{v}}, \tag{9}$$

$$\mathbf{D}_s = \langle \mathcal{L}_s^{-1} \tilde{\mathbf{a}}_s \tilde{\mathbf{a}}_s \rangle. \tag{10}$$

For the purposes of the present calculation, we presume that $Q(f_s)$ is a given quantity that is in practice calculated from an appropriate RF model. Note in this form, $Q(f_s)$ is written generally as a function of three spatial variables, three velocity space variables and time. Since the specifics of the RF physics is not needed in the ensuing calculation, the bracket notation is dropped in the following.

The fluid equations are derived by taking moments of the kinetic equation. These are given by

$$\frac{\partial n_s}{\partial t} + \nabla \cdot (n_s \mathbf{V}_s) = 0, \tag{11}$$

$$m_s n_s (\frac{\partial \mathbf{V}_s}{\partial t} + \mathbf{V}_s \cdot \nabla \mathbf{V}_s) = n_s q_s (\mathbf{E} + \mathbf{V}_s \times \mathbf{B}) - \nabla p_s - \nabla \cdot \pi_s + \mathbf{R}_s + \mathbf{F}_{s0}^{rf}, \tag{12}$$

$$\frac{3}{2} n_s (\frac{\partial T_s}{\partial t} + \mathbf{V}_s \cdot \nabla n_s) + n_s T_s \nabla \cdot \mathbf{V}_s = -\nabla \cdot \mathbf{q}_s - \pi_s : \nabla \mathbf{V}_s + Q_s + S_{s0}^{rf}, \tag{13}$$

The additional force and energy input terms due to the RF are given by

$$\mathbf{F}_{s0}^{rf} = \int d^3\mathbf{v} m_s \mathbf{v}' Q(f_s), \tag{14}$$

$$S_{s0}^{rf} = \int d^3\mathbf{v} \frac{1}{2} m_s v'^2 Q(f_s), \tag{15}$$

where $\mathbf{v}' = \mathbf{v} - \mathbf{V}_s$. The RF is assumed to produce no particles

$$\int d^3\mathbf{v} Q(f) = \int d^3\mathbf{v} \frac{\partial}{\partial \mathbf{v}} \cdot \mathbf{J}_s^{v/rf} = 0, \tag{16}$$

and as such does not modify the density evolution equation.

While the fluid equations are exact, a treatment of the closure problem is needed. In particular, expressions for the heat flux

$$\mathbf{q}_s = \int d^3\mathbf{v} \frac{1}{2} m_s v'^2 \mathbf{v}' f_s(\mathbf{v}), \tag{17}$$

and stress tensor

$$\pi_s = \int d^3\mathbf{v} m_s \mathbf{v}' \mathbf{v}' f_s(\mathbf{v}), \tag{18}$$

are required to close the system of equations. In addition to providing terms in the fluid equations, the RF will also modify the closures.

III. CHAPMAN-ENSKOG CLOSURE PROCEDURE

Since the problem of interest is primarily a modification to Ohm's law (for localized current drive), the closest analogy is with the classic Spitzer problem which is solved via a perturbation theory assuming a small parallel electric field relative to the Dreicer field. Assuming the parallel component of $\mathbf{F}_{s0}^{rf}$ is comparable to $n_s q_s \mathbf{E}$, the RF term can be treated as small. Hence, we imagine that the RF-induced deviation to the distribution function is "small" and a perturbation theory can be employed. We write,

$$f_s = f_{Ms} + F_s, \tag{19}$$

where $|F_s| \ll f_{Ms}$ and f_{Ms} denotes a flow shifted Maxwellian distribution function for species s. This approximation is quite reasonable for ECCD, but not a good approximation for other forms of RF heating and current drive. Nonetheless, the approximation allows for analytic progress on the closure problem with RF. The process is to now derive an equation for the kinetic distortion F_s whose solution can be used to solve for the closure quantities $\mathbf{q}_s$ and π_s.

By assuming a lowest order Maxwellian, the additional RF sources in the fluid equations can generally be written in terms of fluid variables. With $f_s \approx f_{Ms}$, we have

$$\mathbf{F}_{s0}^{rf} = \int d^3\mathbf{v} m_s \mathbf{v}' Q(f_s) = \int d^3\mathbf{v} m_s \mathbf{v}' Q(f_{Ms}), \tag{20}$$

$$S_{s0}^{rf} = \int d^3\mathbf{v} \frac{1}{2} m_s v'^2 Q(f_s) = \int d^3\mathbf{v} \frac{1}{2} m_s v'^2 Q(f_{Ms}). \tag{21}$$

The quantities $\mathbf{F}_{s0}^{rf}$ and S_{s0}^{rf} are now expressed as functions of low order fluid moments. Once the RF quasilinear diffusion operator is specified, the new source terms in the fluid equations are determined as functions of the species' density, temperature and fluid velocity. However, this does not complete the calculation as RF contributions also modify the closure moments as well.

Using a Chapman-Enskog-like approach, a kinetic equation for the kinetic distortion is derived with RF source terms. The leading order Maxwellian,

$f_{Ms} = n_s(m/2\pi T_s)^{3/2} e^{-mv'^2/2T_s}$ is a function of the fluid variables $n_s(\mathbf{x},t)$, $T_s(\mathbf{x},t)$ and $\mathbf{V}_s(\mathbf{x},t)$ that satisfy the fluid equations. The Chapman-Enskog ansatz is that the kinetic distortion F_s has no density, temperature or momentum moments.

$$\int d^3\mathbf{v} F_s = \int d^3\mathbf{v} m_s \mathbf{v}' F_s = \int d^3\mathbf{v} \frac{1}{2} m_s v'^2 F_s = 0. \tag{22}$$

Using the Chapman-Enskog approach and the perturbation scheme implied by the modified Spitzer problem, we have

$$\frac{df_s}{dt} = C(f_s) + Q(f_s) \rightarrow \frac{dF_s}{dt} - C(f_s) = -\frac{df_{Ms}}{dt} + Q(f_{Ms}), \tag{23}$$

which is simplified by using the fluid equations to evaluate df_{Ms}/dt:

$$\begin{aligned} \frac{dF_s}{dt} - C(f_s) &= Q(f_{Ms}) + \mathbf{v}' \cdot [\nabla \cdot \pi_s - \mathbf{R}_s - \mathbf{F}^{rf}_{s0}] \frac{f_{M_s}}{n_s T_s} \\ &+ (\frac{m_s v'^2}{3T_s} - 1)[\pi_s : \nabla \mathbf{V}_s + \nabla \cdot \mathbf{q}_s - Q_s - S^{rf}_{s0}] \frac{f_{Ms}}{n_s T_s} \\ &-(\frac{m_s v'^2}{2T_s} - \frac{5}{2}) \mathbf{v}' \cdot \nabla T_s \frac{f_{Ms}}{T_s} + \frac{m_s}{T_s}[\mathbf{v}'\mathbf{v}' - \frac{v'^2}{3}\mathbf{I}] : \nabla \mathbf{v}_s f_{Ms}, \end{aligned} \tag{24}$$

where $\mathbf{v}' = \mathbf{v} - \mathbf{V}_s$. This is the governing equation for the kinetic distortion. Notice, that in this formulation, the effect of the RF enters solely as additional source terms. The power of this approach is while there is no general procedure for inverting the linear operator on F_s there are various analytic and numerical techniques available to solve this problem [17, 18]. Using the procedure outlined in that work, we can rely on these techniques to solve the current problem of interest as well since the effect of the RF enters as a source term in the equation for the kinetic distortion.

Since equilibration along field lines is of primary interest to the ECCD stabilization problem, a particularly interesting limit asserts the dominance of the free streaming term in the kinetic equation. In this limit, Eq. (24) has the form

$$v_{||} \nabla_{||} F_s - C(f_s) = Sources, \tag{25}$$

where the sources on the right correspond to contributions from temperature and flow gradients, closure moments and the RF sources. Procedures have been formulated for inverting this operator for evolving magnetic fields that result in integral expressions for closure moments. Whereas the primary application of Refs. [17] and [18] was the calculation of heat flux in the presence of temperature gradients in the vicinity on magnetic islands, it is clear that the procedure outlined in those papers can also be used for the RF coupling problem as additional source terms need only be added.

IV. MODIFIED SPITZER PROBLEM

While a general solution to Eq. (24) is not available, we can illustrate the procedure to compute the closures for a simplified example. In the following, a steady-state, homogeneous magnetic field is assumed with $\nabla p_s = \nabla T_s = \nabla \mathbf{v}_s = \nabla \cdot \pi = 0$. The kinetic equation reduces to

$$\frac{dF_s}{dt} - C(f_s) = Q(F_{Ms}) - \mathbf{v}' \cdot (\mathbf{R}_s + \mathbf{F}_{s0}^{rf}) \frac{f_{Ms}}{n_s T_s} + ..., \tag{26}$$

where the remaining terms on the right are even in v' and will not affect parallel Ohm's law, the primary modification to the fluid equations in the presence of ECCD.

The general procedure used to solve for the closure problem is to expand the distribution function in an orthogonal basis set. Moments of the kinetic equation are then taken that produce a sequence of linear algebra equations for the coefficients of the distribution function's basis set. The matrix equation is then solved to produce solutions for the higher order fluid moments.

Ignoring electron intertia and assuming $m_e \ll m_i$, the component of Ohm's law parallel to the magnetic field is given by

$$0 = -n_e e E_{||} + R_{||} + F_{||e0}^{rf}, \tag{27}$$

where $R_{||}$ is the collisional friction and $F_{||e0}$ is the parallel component of Eq.

(20). The relevant closure issue for this problem is the determination of the collisional friction.

In this collisional limit, the kinetic equation for each species can be solved by expanding the distribution function in Laguerre polynomials $L_i^{3/2}(x)$.

$$f_s - f_{Ms} = f_{Ms}\frac{2\mathbf{v}\cdot}{v_{Ts}^2}\sum_{i=0}^{\infty}\mathbf{u}_{si}L_i^{3/2}(x) = f_{Ms}\frac{2\mathbf{v}\cdot}{v_{Ts}^2}(\mathbf{V}_s L_0^{3/2} - \frac{2}{5n_s T_s}\mathbf{q}_s L_1^{3/2} + ...) \tag{28}$$

Here, the coefficients of this expansion are the fluid moments $\mathbf{u}_{si}$ and the arguments of the Laguerre polynomials are given by $x = v^2/v_{Ts}^2$ with v_{Ts} the species thermal speed. In the above, $f_{Ms} = n_s(m/2\pi T_s)^{3/2}e^{-mv^2/2T_s}$ is written as the unflow shifted Maxwellian as the fluid flow in the Spitzer problem is small relative to the thermal velocity; the contribution $e^{-mv'^2/2T_s} - e^{-mv^2/2T_s} \approx 2\mathbf{V}_s\cdot\mathbf{v}/v_{Ts}^2 e^{-mv^2/2T_s}$ corresponds to the first term in the expansion. Moments of the kinetic equation are now taken using

$$\int d^3\mathbf{v} m_e v_{||} L_j^{3/2}(x)[C(f_s) = -Q(F_{Ms}) + \mathbf{v}'\cdot(\mathbf{R}_s + \mathbf{F}_{s0}^{rf})\frac{f_{Ms}}{n_s T_s}], \tag{29}$$

Using Eq. (27) and known properties of the electron-electron and electron-ion collision operators, a sequence of algebraic expressions are derived given by

$$n_e m_e \nu_e \sum_k (L_{jk}^e u_{ek} + L_{jk}^i u_{ik}) = -n_e e E_{||}\delta_{j,0} + F_{||je}^{rf}, \tag{30}$$

where $\delta_{i,j}$ is the Kronecker-delta, ν_e is the electron collision frequency, a single ion species is assumed and moments of the RF operator are

$$\mathbf{R}_{je}^{rf} \equiv \int d^3\mathbf{v}\mathbf{v} L_j^{3/2}(x) Q(f_{Me}). \tag{31}$$

The matrix coefficients L_{jk}^e and L_{jk}^i are known dimensionless numbers that depend only on the ion charge Z [20]. Further, momentum conservation properties of the collision operator require $L_{j0}^i = -L_{j0}^e$ and $L_{jk}^i = 0$ for

$k \neq 0$. As such Eq. (30) has the form

$$n_e m_e \nu_e \begin{pmatrix} L^e_{00} & L^e_{01} & L^e_{02} & \cdots \\ L^e_{10} & L^e_{11} & L^e_{12} & \cdots \\ L^e_{20} & L^e_{21} & L^e_{22} & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{pmatrix} \begin{pmatrix} u_{||0e} - u_{||0i} \\ u_{||1e} \\ u_{||2e} \\ \cdots \end{pmatrix} = - \begin{pmatrix} n_e e E_{||} \\ 0 \\ 0 \\ \cdots \end{pmatrix} + \begin{pmatrix} F^{rf}_{||0e} \\ F^{rf}_{||1e} \\ F^{rf}_{||2e} \\ \cdots \end{pmatrix}. \tag{32}$$

Inverting the matrix $L^{-1} = \Lambda$ and linear algebra allows one to express the parallel collisional friction as

$$R_{||} = \frac{m_e \nu_e}{e \Lambda_{00}} J_{||} + \sum_{k=1} \frac{\Lambda_{0k}}{\Lambda_{00}} F_{||ek}, \tag{33}$$

and parallel Ohm's law takes the form

$$E_{||} = \eta_{sp} J_{||} + \sum_{k=0} \frac{\Lambda_{0k}}{\Lambda_{00}} \frac{F^{rf}_{||ek}}{n_e e}, \tag{34}$$

where $\eta_{sp} = m_e \nu_e / n_e e^2 \Lambda_{00}$ is the Spitzer resistivity [20]. The external RF source has modified the closure moment and produced additional terms in the parallel Ohm's law beyond the simple inclusion of the $\mathbf{F}^{rf}_{e0}$ in the fluid momentum balance equation.

V. DISCUSSION

A theory has been developed that allows for the inclusion of externally applied RF sources into the fluid equations. This procedure is viable for situations where the RF introduces small non-Maxwellian distortions of the distribution function. The fluid moment equations are derived from a kinetic equation that includes the effect of RF sources. The source term of the kinetic equation generally introduces additional terms in the fluid equations. The RF source also modifies the closure moments and a complete description of the dynamics requires a procedure for calculating this modification.

The closure problem is formulated using a Chapman-Enskog like approach where the distribution function is written as the sum of a lowest order flow shifted Maxwellian and a small kinetic distortion. An equation for the kinetic distortion is derived and given in Eq. (24); this is the primary result of this work. The important observation from this procedure is that the effect of the RF shows up as modifying the 'source' term for the kinetic distortion equation. Hence, calculations of modifications to the fluid closure moments from the RF fields closely resembles prior work using a Chapman-Enskog-like formulation to calculate heat fluxes and viscous stresses [17, 18]. Additionally, we note that since one of the primary applications for this procedure is for numerical simulations of NTM stabilization in the presence of ECCD, a fluid theory approach is viable as electron cyclotron wave heating and current drive do not produce a distribution function that is radically different from a lowest order Maxwellian. This is not the case for all forms of external RF sources; for example, lower hybrid waves produce a highly non-Maxwellian distribution function.

The derived kinetic equation for the kinetic distortion in the presence of an RF source can be solved in the simple limit of steady-state homogeneous magnetic fields with no viscosity, pressure, temperature or flow velocity gradients. The resultant problem resembles the classic Spitzer problem and produces a modified fluid closure moment for the collisional friction given in Eq. (33). A sequence of extensions to the modified Spitzer can be developed. These include calculations for bumpy cylinders, toroidal equilbria, the inclusion of time-dependent processes, multiple length scales calculations, the effect of magnetic islands, etc. This is left as future work.

Finally, we comment on how this work can be employed in computation models of RF-MHD coupling. The primary requirement for the RF codes is to provide $Q(f_{Ms})$ as a function of the six phase space variables and time given appropriate information from the extended MHD code. This quantity is subsequently employed to produce the additional terms in the fluid moment equations, $\mathbf{R}_{s0}^{RF}$ and S_{s0}^{rf} and to be used in the calculation the kinetic distortion and subsequent calculation of the closure moments. The results of these calculations are then passed back to the fluid codes.

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