Key hypothesis of paleoclassical model

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The paleoclassical model of radial electron heat transport in resistive, current-carrying toroidal plasmas is based on a key hypothesis — that electron guiding centers move and diffuse with radially localized annuli of poloidal magnetic flux. This hypothesis is shown to result from the temporal evolution of the guiding center version of the canonical toroidal angular momentum on the magnetic field diffusion time scale $\tau_\eta \equiv a^2/6D_\eta$. These guiding center motion and diffusion effects modify the usual drift-kinetic equation by adding a τ_η time-scale Fokker-Planck spatial diffusion operator.

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Recently, a new "paleoclassical" model for cross-field, radial electron heat transport in current-carrying, resistive toroidal plasmas has been developed and published: fundamental paleoclassical concepts in a sheared slab model [1], comprehensive axisymmetric toroidal magnetic field paleoclassical model [2], and brief summary of the model plus preliminary interpretations of various experimental results with it [3]. More extensive comparisons with experimental data [4] have demonstrated that the irreducible minimum level of radial electron heat transport is often set by the paleoclassical model.

The key hypothesis of the paleoclassical model is that [1–3] in resistive, current-carrying toroidal plasmas electron guiding centers move and diffuse along with and just as small radially localized bundles of the poloidal magnetic flux do on the magnetic ("skin") diffusion time scale. In particular, it is hypothesized that a Fokker-Planck-type spatial diffusion operator representing these advection and diffusion effects should be added to the right side of the usual drift-kinetic equation [5]. While this key hypothesis was motivated phenomenologically [2], it was not derived from "first principles." This note shows that this hypothesis can be justified [6] via a multiple-time-scale analysis of the evolution of the guiding center version of the canonical toroidal angular momentum on the magnetic field diffusion time scale.

For an axisymmetric magnetic field described by $\mathbf{B} = I\boldsymbol{\nabla}\zeta + \boldsymbol{\nabla}\zeta\times\boldsymbol{\nabla}\psi_p \equiv \mathbf{B}_t + \mathbf{B}_p$ and a flux-surface-averaged [$\langle A(\mathbf{x})\rangle$] parallel Ohm's law, evolution equations for the toroidal magnetic flux ψ_t and the poloidal magnetic flux ψ_p are [2, 7, 8] (neglecting electron inertia effects)

$$\frac{d\psi_t}{dt} \equiv \left.\frac{\partial\psi_t}{\partial t}\right|_{\mathbf{x}} + \bar{u}_G\frac{\partial\psi_t}{\partial\rho} = 0, \qquad (1)$$

$$\frac{d\psi_p}{dt} \equiv D_\eta\,\Delta^+\psi_p - S_\psi, \quad D_\eta \equiv \frac{\eta_\parallel^{\rm nc}}{\mu_0}. \qquad (2)$$

Here, $\bar{u}_G \equiv \langle\mathbf{E}{\cdot}\mathbf{B}_p\rangle/\psi_p' I\langle R^{-2}\rangle$ is the "grid velocity" (subsript G, relative to fixed laboratory coordinates) of the toroidal magnetic flux ψ_t, D_η is the magnetic field diffusivity induced by the neoclassical parallel resistivity $\eta_\parallel^{\rm nc}$ [2], and Δ^+ is a cylindrical-like second order differential operator in the radial direction [2, 7, 8]. The source of poloidal magnetic flux is $S_\psi \equiv \partial\Psi/\partial t - (\mu_e/\nu_e)(\eta_0/\langle R^{-2}\rangle)dP/d\psi_p + \eta_\parallel^{\rm nc}\langle\mathbf{J}_S{\cdot}\mathbf{B}\rangle/I\langle R^{-2}\rangle$, which includes sources of poloidal magnetic flux due to flux changes in the central solenoid (ohmic transformer), and parallel currents due to the bootstrap current and a non-inductive current source $\mathbf{J}_S$ [2].

From (1) one sees that the toroidal magnetic flux ψ_t is stationary, except for the radial grid velocity $\bar{u}_G$, which, for steady-state conditions, can be shown [9] to be $\mathcal{O}(\epsilon^2, B_p^2/B_t^2)$ smaller than the terms on the right of (2) for a torus of large aspect ratio ($\epsilon \equiv r/R_0 << 1$) where the poloidal magnetic field B_p is smaller than the toroidal field B_t. Henceforth, it will be assumed that the grid velocity $\bar{u}_G$ is negligible or accounted for explicitly by carrying out subsequent calculations in the Lagrangian frame of ψ_t; thus, the toroidal flux ψ_t will be a stationary frame of reference. Because the toroidal magnetic flux is essentially stationary in toroidal plasmas, it is convenient to define a dimensionless, cylindrical-type radial variable ρ:

$$\rho = \sqrt{\psi_t\,/\psi_t(a)} \simeq r/a, \quad \text{radial coordinate.} \qquad (3)$$

Neoclassical transport fluxes are determined [2, 7–9] in the rest frame of the toroidal magnetic flux ψ_t; thus, ρ is normally used as the most appropriate radial coordinate for modeling transport in evolving toroidal plasmas.

From (2) one sees that $D_\eta \propto \eta$ causes the poloidal magnetic flux ψ_p to diffuse relative to the stationary toroidal magnetic flux ψ_t, and hence ρ. Also, when $d\psi_p/dt \neq 0$, it moves relative to ψ_t, ρ. In the usual drift-kinetic equation for an axisymmetric toroidal plasma [5], the radial guiding center coordinate is identified with the local value of the poloidal magnetic flux ψ_p. In temporally evolving situations where $d\psi_p/dt \neq 0$ there is a clear problem with using the (changing) poloidal flux ψ_p as the radial coordinate. Furthermore, the radial diffusion of electron guiding centers tied to diffusing thin annuli of poloidal flux ψ_p (small bundles of field lines) — see (13) below — is usually not taken into account. These problems will be remedied by changing the radial variable

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in the drift-kinetic equation from ψ_p to the essentially stationary, toroidal-flux-based coordinate ρ and adding a Fokker-Planck spatial diffusion operator to the usual drift-kinetic equation.

To illustrate the physical problem with using a changing poloidal flux ψ_p as a radial coordinate, consider the following question: when $d\psi_p/dt \neq 0$, on what ψ_p and ψ_t, ρ will an electron be after one gyroperiod $\tau \equiv 2\pi/\omega_{ce}$ ($\omega_{ce} \equiv q_e B/m_e$, electron gyrofrequency). Physically, after one gyroperiod an electron will return to its original radial position, to within second order gyroradius correction terms. Since the toroidal flux did not change in the time τ, the electron will return to its original ψ_t, ρ position. However, since ψ_p changed during this time, it will now be on a different poloidal flux surface ψ_p. To determine how far the ψ_p surface it was originally on is displaced radially in the time τ, expand ψ_p in a Taylor series about an arbitrary initial radial position ρ_0,

$$\psi_p(\rho,t) = \psi_0(\rho_0,t) + x\,\psi_p' + \cdots, \quad \psi_p' \equiv \partial\psi_p/\partial\rho|_{\rho_0}, \quad (4)$$

in which x is a small radial distance ($|x| << 1$),

$$x \equiv \rho - \rho_0, \quad \text{dimensionless local radial coodinate.} \quad (5)$$

To determine how far the original ψ_p surface moved in the time τ, take the time derivative of (4) and set it to zero. Then, using (2) to evaluate the time derivative of ψ_0 and integrating over the short time τ, one obtains

$$\bar{u}_{\dot\psi} \equiv dx_\psi/dt = -\,\dot\psi/\psi_p' \quad \Longrightarrow \quad \delta x_\psi = -\,\tau\,\dot\psi/\psi_p'. \quad (6)$$

Here, the rate of change in ψ_p has been defined by

$$\dot\psi = D_\eta\,\Delta_0^+\psi_0 - S_\psi, \quad (7)$$

in which the subscript 0 on the differential operator Δ_0^+ means differentiations are taken with respect to ρ_0.

The last result in (6) shows that when $\dot\psi \neq 0$ the ψ_p surface moves a distance δx_ψ in the time τ. Thus, if a temporally changing ψ_p is used as the radial coordinate, the electron's position after one gyroperiod will have to be corrected by changing the position x_g of its guiding center (subscript g) by $\delta x_g = -\,\delta x_\psi = -\tau\bar{u}_{\dot\psi} = \tau\dot\psi/\psi_p'$. Clearly, a better procedure would be to adopt a radial coordinate based on the stationary toroidal flux, i.e., ρ — because the electron always returns to the same ρ after a gyroperiod, even when $\dot\psi \neq 0$.

The usual drift-kinetic equation [5] uses the poloidal flux ψ_p as a radial coordinate. To see why this is done, consider the $\mathbf{e}_\zeta \equiv R^2\boldsymbol{\nabla}\zeta$ (covariant base vector) projection of Newton's second law with a Lorentz force $\mathbf{F} = q_e(\mathbf{E}+\mathbf{v}\times\mathbf{B})$, and use $\mathbf{e}_\zeta\cdot\mathbf{E} = -\,\mathbf{e}_\zeta\cdot\partial\mathbf{A}/\partial t = \partial\psi_p/\partial t$ (i.e., neglect the scalar potential ϕ), to obtain [2]

$$\frac{d}{dt}(m_e\mathbf{e}_\zeta\cdot\mathbf{v}) = q_e\left(\frac{\partial\psi_p}{\partial t} + (\mathbf{v}\cdot\boldsymbol{\nabla}\rho)\psi_p'\right) = q_e\frac{d\psi_p}{dt}, \quad (8)$$

which is valid on the gyromotion and longer time scales. Normally one simply integrates this equation over time to obtain the constancy of the canonical toroidal angular momentum: $p_\zeta = m_e\mathbf{e}_\zeta\cdot\mathbf{v} - q_e\psi_p = \text{constant}$. Drift-kinetic theory [5] usually uses this constant of the particle motion, or its lowest order in small gyroradius form $p_\zeta \simeq -q_e\psi_p$ and hence just ψ_p, as its radial coordinate.

A multiple-time-scale analysis will be used to develop a procedure for determining the evolution of p_ζ, and hence the mean electron guiding center position, on the (long) magnetic field diffusion time scale. On this time scale p_ζ is not constant because in resistive, current-carrying toroidal plasmas the poloidal magnetic field $\mathbf{B}_p$ is a driven-dissipative system — from (2) its equilibrium results from balancing its source S_ψ against the magnetic field diffusion $D_\eta\,\Delta^+\psi_p$. Thus, the poloidal magnetic field $\mathbf{B}_p$ is not a conservative field on the magnetic field diffusion time scale; hence, a Hamiltonian description is not applicable on this (long) time scale.

The usual gyromotion plus guiding center equations for the particle velocity $\mathbf{v}$ [5] can be written concisely as

$$\mathbf{v} = \mathbf{v}_g + \tilde{\mathbf{v}}_\perp, \quad \mathbf{v}_g \equiv \frac{d\mathbf{x}_g}{dt} = \frac{v_\parallel\mathbf{B}}{B} + \mathbf{v}_D, \quad (9)$$

in which $\mathbf{v}_g$ and $\mathbf{x}_g$ are the electron's guiding center velocity and position, $\tilde{\mathbf{v}}_\perp$ is its gyromotion velocity whose average over a gyroperiod vanishes, $v_\parallel \equiv \mathbf{B}\cdot\mathbf{v}/B$ is its speed parallel to $\mathbf{B}$, and $\mathbf{v}_D$ represents its drift velocity, which is one order smaller (than $v_\parallel$) in the small gyroradius expansion ($v_\perp/\omega_{ce}L_\perp << 1$) and will be neglected henceforth. Substituting $\mathbf{v}$ from (9) into (8) and averaging over a gyroperiod to eliminate the gyromotion terms, one obtains p_ζ on the guiding center motion time scale (i.e., on time scales longer than the gyroperiod τ):

$$\frac{dp_{\zeta g}}{dt} = 0, \quad p_{\zeta g} \equiv m_e\frac{v_\parallel I}{B} - q_e x_g\psi_p' = \text{constant}, \quad (10)$$

in which $x_g \equiv \mathbf{x}_g\cdot\boldsymbol{\nabla}\rho = \rho_g - \rho_0$ is the (assumed small) dimensionless radial distance of the guiding center from the ρ_0 surface. Defining $p_{\zeta g} = -\,q_e(\psi_p - \psi_0)$ to be consistent with the Taylor series expansion in (4), the poloidal flux at the electron guiding center position x_g becomes

$$\psi_p(\rho_g) = \psi_0(\rho_0) + [x_g(\rho,t) - (m_e/q_e)(v_\parallel I/B\psi_p')]\psi_p'. \quad (11)$$

This result shows that an electron guiding center is "tied to" the radially localized, thin annulus of poloidal magnetic flux $\delta\psi_g \equiv \psi_p(\rho_g) - \psi_0(\rho_0) << \psi_0$. The evolution of this small amount of magnetic flux is governed by a diffusion equation ($d\,\delta\psi_g/dt \simeq D_\eta\,\Delta^+\delta\psi_g$) and characterized by [2] Fokker-Planck advection and diffusion coefficients $\langle\Delta x_\psi\rangle/\Delta t = \bar{u}_G + \bar{u}_{\dot\psi}$ and $\langle(\Delta x_\psi)^2\rangle/\Delta t = 2\,\bar{D}_\eta$. Since the electron guiding center is tied to this thin annulus of poloidal flux, one can expect the electron guiding center x_g to move and diffuse radially along with it.

When ψ_p is changing in time (typically on the magnetic diffusion time scale), one needs to allow ψ_p, ψ_0 and x_g to depend on this longer time scale as well: thus, one has $\psi_p(\rho_g, t_\eta)$, $\psi_0(\rho_0, t_\eta)$ and $x_g(\rho, t, t_\eta) = \bar{x}_g(\rho, t_\eta) + \breve{x}_g(\rho, t, t_\eta)$, in which t is the guiding center motion time scale, $t_\eta \lesssim \tau_\eta \equiv a^2/6D_\eta$ is the magnetic "skin" diffusion time scale, $\breve{x}_g$ represents the variation of the guiding center position (via bounce motion) in x_g described by (11), and $\bar{x}_g$ represents the mean position of the electron guiding center. Taking the total time derivative of $p_{\zeta g}$ with respect to t_η, one obtains

$$\frac{dp_{\zeta g}(x_g, t_\eta)}{dt_\eta} = \frac{\partial p_{\zeta g}}{\partial t_\eta}\bigg|_{x_g} + \frac{\partial p_{\zeta g}}{\partial x_g}\bigg|_{t_\eta} \frac{dx_g}{dt_\eta}. \quad (12)$$

From (10), one sees $p_{\zeta g}$ should not depend explicitly on time. But since it may change on the t_η time scale due to motion of the guiding center x_g, one has $\partial p_{\zeta g}/\partial t_\eta = 0$ and $dp_{\zeta g}/dt_\eta = -q_e(dx_g/dt_\eta)\psi_p'$. Further, using (2) and (7) one has $dp_{\zeta g}/dt_\eta = -q_e(d\psi_g/dt_\eta - d\psi_0/dt_\eta) = -q_e[(\dot{\psi} + D_\eta\, \Delta^+ x_g) - \dot{\psi}]\psi_p' = -q_e\psi_p' D_\eta\, \Delta^+ x_g$. Thus, requiring no secular growth in $\breve{x}_g$, (12) simplifies to

$$\frac{d\bar{x}_g(\rho, t_\eta)}{dt_\eta} = D_\eta\, \Delta^+ \bar{x}_g. \quad (13)$$

This result can also be obtained directly by taking the t_η derivative of (11) using the definitions of ψ_p, ψ_0 and x_g on the t_η time scale stated earlier in this paragraph, and averaging over the bounce motion in $\breve{x}_g$.

Equation (13) is a diffusion equation for the mean guiding center position $\bar{x}_g(\rho, t_\eta)$ on the magnetic field diffusion time scale t_η. For a mean guiding center position initially at ρ_{g0} [i.e., $\bar{x}_g(\rho, 0) = \bar{x}_{g0}\delta(\rho - \rho_{g0})$] and short times ($t_\eta << \tau_\eta$), its radially localized solution for which $\Delta^+\bar{x}_g \simeq (1/\bar{a}^2)\,\partial^2\bar{x}_g/\partial x^2$ in which $\bar{a} \simeq a$ is an average minor radius defined by $1/\bar{a}^2 \equiv \langle|\boldsymbol{\nabla}\rho|^2/R^2\rangle/\langle R^{-2}\rangle$ [2], yields a standard diffusive response (for $t_\eta \geq 0$):

$$\bar{x}_g(x, t_\eta) = \bar{x}_{g0}\, \frac{e^{-(x-\bar{x}_{g0})^2/4\bar{D}_\eta t_\eta}}{(4\pi\bar{D}_\eta t_\eta)^{1/2}}, \quad \bar{D}_\eta \equiv \frac{D_\eta}{\bar{a}^2}. \quad (14)$$

Thus, the mean position of the electron guiding center diffuses on the magnetic field diffusion time scale with a diffusion coefficient $\bar{D}_\eta$. Reintroducing the ψ_t, ρ grid speed effects due to $\bar{u}_G$, Fokker-Planck coefficients which represent the grid motion and diffusion effects are [2, 10]

$$\frac{\langle\Delta x_g\rangle}{\Delta t} = \bar{u}_G, \quad \frac{\langle(\Delta x_g)^2\rangle}{\Delta t} = 2\,\bar{D}_\eta. \quad (15)$$

Because the magnetic geometry is complicated, the Fokker-Planck coefficients should be written in a general vectorial form. In terms of the covariant base vector in the radial direction $\mathbf{e}_\rho \equiv \partial\mathbf{x}/\partial\rho = \sqrt{g}\,\boldsymbol{\nabla}\theta\times\boldsymbol{\nabla}\zeta$ with $\sqrt{g} \equiv 1/\boldsymbol{\nabla}\rho\boldsymbol{\cdot}\boldsymbol{\nabla}\theta\times\boldsymbol{\nabla}\zeta$ for which $\mathbf{e}_\rho\boldsymbol{\cdot}\boldsymbol{\nabla}\rho = 1$, they are [2]:

$$\frac{\langle\Delta\mathbf{x}_g\rangle}{\Delta t} \equiv \frac{\langle\Delta x_g\rangle}{\Delta t}\mathbf{e}_\rho, \quad \frac{\langle\Delta\mathbf{x}_g\,\Delta\mathbf{x}_g\rangle}{\Delta t} \equiv \frac{\langle(\Delta x_g)^2\rangle}{\Delta t}\mathbf{e}_\rho\mathbf{e}_\rho. \quad (16)$$

Since these Fokker-Planck coefficients are essentially the same [10] as those for a radially localized poloidal magnetic flux [2], these results explicitly justify the key paleclassical hypothesis that electrons advect and diffuse with the thin annulus of poloidal magnetic flux $\delta\psi_g$. In principle these coefficients are valid for time scales $\tau << \Delta t << \tau_\eta$. However, since the equilibrium parallel Ohm's law and (2) are only valid [2] on time scales longer than the electron collision time $1/\nu_e$, for these Fokker-Planck coefficients one has

$$1/\nu_e < \Delta t << \tau_\eta \equiv a^2/6D_\eta, \quad \text{validity time scale.} \quad (17)$$

The usual ψ_p-based drift-kinetic equation (DKE), correct through first order in the small gyroradius expansion, can be written as [5]

$$\frac{\partial f}{\partial t} + \frac{d\mathbf{x}_g}{dt}\boldsymbol{\cdot}\boldsymbol{\nabla} f + \dot{\varepsilon}_g\frac{\partial f}{\partial\varepsilon_g} = \mathcal{C}\{f\}, \quad \text{usual DKE}, \quad (18)$$

in which the guiding center distribution function $f = f(\psi_p, \theta_g, \zeta_g, \varepsilon_g, \mu, t)$, the guiding center velocity $d\mathbf{x}_g/dt \equiv \mathbf{v}_g$ is defined in (9), the particle guiding center kinetic energy $\varepsilon_g \equiv mv_\parallel^2/2 + \mu B$ with magnetic moment $\mu \equiv mv_\perp^2/2B(\mathbf{x}_g, t)$, $\dot{\varepsilon}_g = \mu\,\partial B/\partial t + q_e\mathbf{v}_g\boldsymbol{\cdot}\mathbf{E}$, and $\mathcal{C}\{f\}$ is the Fokker-Planck Coulomb collision operator.

Changing from the ψ_p radial coordinate to the nearly stationary toroidal-magnetic-flux-based radial coordinate ρ and taking account of the grid motion and diffusion (in ρ) of the mean position of the electron guiding centers by adding a Fokker-Planck spatial diffusion operator to the usual DKE yields the magnetic-field-diffusion-modified drift-kinetic equation (MDKE) [2]:

$$\frac{\partial f}{\partial t} + \frac{d\mathbf{x}_g}{dt}\boldsymbol{\cdot}\boldsymbol{\nabla} f + \dot{\varepsilon}_g\frac{\partial f}{\partial\varepsilon_g} = \mathcal{C}\{f\} + \mathcal{D}\{f\}, \quad \text{MDKE}, \quad (19)$$

in which $f = f(\rho_g, \theta_g, \zeta_g, \varepsilon_g, \mu, t)$. Here, the grid motion and diffusion of the electron guiding centers is taken into account via the Fokker-Planck spatial diffusion operator

$$\mathcal{D}\{f\} \equiv -\boldsymbol{\nabla}\boldsymbol{\cdot}\frac{\langle\Delta\mathbf{x}_g\rangle}{\Delta t}f + \frac{1}{2}\boldsymbol{\nabla}\boldsymbol{\cdot}\left(\boldsymbol{\nabla}\boldsymbol{\cdot}\frac{\langle\Delta\mathbf{x}_g\,\Delta\mathbf{x}_g\rangle}{\Delta t}f\right). \quad (20)$$

For distributions $f(\rho)$ whose spatial dependence is solely through dependence on the radial coordinate ρ, the flux-surface-average of this Fokker-Planck spatial diffusion operator is approximately [2]

$$\langle\mathcal{D}\{f(\rho)\}\rangle \simeq \frac{1}{V'}\frac{\partial}{\partial\rho}\left(-V'\bar{u}_G f + \frac{\partial}{\partial\rho}(V'\bar{D}_\eta f)\right), \quad (21)$$

in which $\mathcal{O}\{\epsilon^2, B_p^2/B_t^2\}$ terms have been neglected and $V' \equiv dV/d\rho$ where $V(\rho)$ is the ρ flux surface volume.

The diffusive part of the Fokker-Planck operator $\mathcal{D}\{f\}$ should be added to the usual drift-kinetic-equation for any resistive, current-carrying plasma for which, since $\eta\,\langle\mathbf{J}\boldsymbol{\cdot}\mathbf{B}\rangle \propto \eta\,\Delta^+\psi_p$, the poloidal magnetic flux and hence electron guiding centers are diffusing radially. [For quasisymmetric stellarator configurations which have both vacuum (ψ_V) and current-driven (ψ_J) "poloidal" magnetic fluxes, the diffusion coefficient D_η should be multiplied by [2] $\psi_J'/(\psi_V' + \psi_J')$ since one then has $dp_{\zeta g}/dt_\eta \propto (d\bar{x}_g/dt_\eta)\,(\psi_V' + \psi_J')$ but $d(\psi_g - \psi_0)/dt_\eta \propto D_\eta\,\Delta^+\bar{x}_g\psi_J'$.]

If ψ_p is used as the radial coordinate in the MDKE, the advective Fokker-Planck coefficient $\langle \Delta x_g \rangle / \Delta t$ in (15) changes to $\bar{u}_G - \bar{u}_{\dot{\psi}}$ [10] — so an electron will be at the same ψ_t, ρ after one gyroperiod, as discussed in the paragraph after (7). This also changes $\bar{u}_G$ to $\bar{u}_G - u_{\dot{\psi}}$ in (21). However, in stationary situations, $\dot{\psi} = 0$ and one has $\bar{u}_{\dot{\psi}} = 0$; then, (19)–(21) define the appropriate MDKE with either ρ or ψ_p as the radial coordinate.

While the MDKE was derived here for electrons, it applies equally well to ions for the time scales indicated in (17), as long as the usual finite gyroradius expansion is applicable. The $\mathcal{D}\{f\}$ Fokker-Planck spatial diffusion operator should also be added to the right of the usual gyrokinetic equations [5]. Since $D_\eta \sim \nu_e (c/\omega_p)^2$ is second order in the electromagnetic skin depth c/ω_p ($\simeq 0.1$ cm for $n_e \simeq 3\times 10^{19}$ m^{-3}), addition of $\mathcal{D}\{f\}$ to the electron and ion drift-kinetic and gyrokinetic equations does not significantly modify plasma properties at zeroth order (density, temperature, pressure) or first order (flows, currents, heat flows along and within flux surfaces) in the small gyroradius, c/ω_p expansion. However, it will affect second order processes; in particular, it leads to paleoclassical radial electron heat transport [1–4]. Also, for perturbations that are radially localized, the $\mathcal{D}\{f\}$ operator introduces a new dissipative process with a rate of order $\nu_e (k_x c/\omega_p)^2$. Such paleoclassical processes could be more important than second order finite ion gyroradius effects where $\varrho_i \equiv v_\perp/\omega_{ci} < c/\omega_p$, which occurs where the local $\beta_i \equiv n_e T_i/(B^2/2\mu_0)$ is less than m_e/m_i (e.g., for $T_i < 300$ eV where $B \sim 2$ T and $n_e \sim 10^{19}$ m^{-3}).

The paleoclassical transport processes introduced via $\mathcal{D}\{f_0\}$ are direct second order secular processes — because they result from charged particle guiding centers being carried with another diffusing quantity, the thin annulus $\delta\psi_g$ of poloidal magnetic flux. Thus, transport operators that result from $\mathcal{D}\{f_0\}$ are not in the usual form of the divergence of radial flows. Rather, as can be seen from (21) with $\bar{u}_G = 0$, they become second derivatives of zeroth order plasma properties (density, temperature); hence, when forced into the usual radial transport flux forms, they embody [2] both diffusive and "pinch-type" (inward flow) density and temperature transport.

Because the $\mathcal{D}\{f_0\}$ Fokker-Planck operator is the same in both the electron and ion drift-kinetic equations, the axisymmetric component [2] of paleoclassical density transport will be the same for both electrons and ions; hence, it will be automatically ambipolar. However, the helically-resonant components that result in the important multiplier [1–3] $M \sim 10$ for paleoclassical transport is probably different for electrons and ions. Thus, the helically-resonant component of paleoclassical transport is not likely to be ambipolar; rather, it probably causes a radial electric field component which, on time scales longer than the poloidal flow damping time scale ($\sim 1/\nu_i$, ion collision time), will lead to a toroidal flow component. In addition, in transient situations the helically resonant components are likely to move radially. Determination of such transient and non-ambipolar paleoclassical processes, and their effects on the radial electric field and toroidal flow velocity, are left for future research.

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