# Resistive Ballooning Modes In HSX?

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This note explores the possibility that resistive ballooning modes [1] could be unstable and responsible for plasma fluctuations and anomalous transport in HSX. The philosophy used is that resistive-interchange theory can be used to estimate key properties of resistive ballooning modes, with suitable adaptations for the local radius of curvature $R_C$ and local magnetic shear length $L_S$. Diamagnetic frequency effects are included since they may be important in HSX plasmas and they determine the wavenumber beyond which one transitions to drift wave instabilities. Theoretical properties of resistive-interchange modes are discussed first. Next, plasma parameters assumed for HSX plasmas are discussed. Finally, the possibility that resistive ballooning modes could be unstable and cause anomalous plasma transport in HSX is considered via adaptations of resistive-interchange analysis. Also, a number of caveats on the simple analysis presented here are noted. The tentative conclusion is that these types of modes could be responsible for $k_y \lesssim 1/$ cm fluctuations and transport observed in HSX near $r/a \simeq 0.7$ where $T_e \simeq 100$ eV — but detailed studies taking account of specifics of HSX geometry and plasma parameters are needed to know for sure.

Using a local simple sheared slab model that includes $\boldsymbol{\nabla} B$ and curvature effects, it can be shown [2] that the dispersion relation for "highly resistive" pressure-gradient modes is given by (see Eq. (29) of [2])

$$\omega(\omega + \tau\omega_*)(\omega - \omega_*) = -\, i\, \nu_\eta\, \omega_A^2 D_I^2 \equiv -\, i\, \gamma_\eta^3, \tag{1}$$

in which (in SI units) $\omega_* \equiv k_y V_* = k_y(T_e/eBL_p) = (k_y \varrho_S)\, c_S/L_p$ (with ion sound speed $c_S \equiv \sqrt{T_e/m_i} = 10^4 \sqrt{T_e(\mathrm{eV})}$ m/s, and $\varrho_S \equiv c_S/\omega_{ci}$) is the electron diamagnetic flow frequency, $1/L_p \equiv -d\ln p_e/dr$, and $\tau = T_i/T_e$. Also,

$$\nu_\eta \equiv k_y^2 \frac{\eta_\|}{\mu_0} = k_y^2 D_\eta, \text{ dissipative frequency}, \quad \omega_A \equiv \frac{c_A}{L_S}, \text{ Alfvén frequency}, \tag{2}$$

in which the magnetic field diffusivity induced by parallel (Spitzer) resistivity is

$$D_\eta \equiv \frac{\eta_\|}{\mu_0} \simeq \frac{700\, Z_{\mathrm{eff}}}{[T_e(\mathrm{eV})]^{3/2}}\, \frac{\mathrm{m}^2}{\mathrm{s}}, \quad \text{magnetic diffusivity}, \tag{3}$$

and the pressure-gradient "drive" for the resistive-interchange instability is

$$D_I \equiv \frac{L_S^2\, \beta}{R_C L_P} \equiv -\frac{L_S^2}{R_C}\frac{d\beta}{dr}, \quad \text{instability drive}, \quad \beta \equiv \frac{p_e + p_i}{B^2/2\mu_0}, \quad \text{total } \beta. \tag{4}$$

Neglecting diamagnetic flow ($\omega_*$) effects, the mode growth rate from (1) is

$$\gamma_\eta \simeq \nu_\eta^{1/3}(D_I \omega_A)^{2/3}, \quad \text{resistive-interchange growth rate}. \tag{5}$$

The radial width of these modes determined from (28) in [2] is

$$\delta \simeq \left[\frac{D_I(\eta_\|/\mu_0)}{\omega - \omega_*}\right]^{1/2} \quad \overset{\gamma_\eta >> \omega_*}{\Longrightarrow} \quad \frac{\delta_\eta}{r} \simeq \frac{D_I^{1/6}}{(k_y r)^{1/3} S^{1/3}}, \quad \text{mode width}. \tag{6}$$

Here, a local ratio of resistive to Alfvén time ($\tau_A \equiv 1/\omega_A$) has been defined:

$$S \equiv \frac{r^2/(\eta_\parallel/\mu_0)}{1/\omega_A} \equiv \frac{\tau_R}{\tau_A} \simeq 1.6 \times 10^{13} \frac{r^2 B [T_e(\mathrm{eV})]^{3/2}}{L_S Z_{\mathrm{eff}} \sqrt{n_i}} \quad \text{Lundquist number.} \tag{7}$$

The diffusion coefficient for plasma transport induced by a turbulent spectrum of such resistive-interchange instabilities is approximately [3]

$$D_{\mathrm{turb}} \simeq \sum_{k_y} \gamma_\eta \delta_\eta^2 \simeq N_{k_y} D_I \frac{\eta_\parallel}{\mu_0} = (N_{k_y} D_I)\, D_\eta, \quad \text{turbulent diffusivity,} \tag{8}$$

in which $N_{k_y}$ is a numerical factor (typically $\sim$ 2–3) that is the logarithm of an effective Reynolds number; it reflects the sum over the unstable mode spectrum [3]. Thus, resistive-interchange mode turbulence induces net transport that is proportional to the magnetic field diffusivity $D_\eta \equiv \eta_\parallel/\mu_0$. Since the Suydam criterion for MHD stability of ideal interchange modes is $D_I < 1/4$, one usually infers that for the smaller $D_I$ in resistive-interchange instabilities $D_{\mathrm{turb}} \lesssim D_\eta$.

Since HSX is typically in a minimum-average-$B$ (average magnetic well) configuration, the average curvature in it is favorable [$R_C < 0$ in (4)] and $D_I < 0$. Hence, ideal and resistive interchange instabilities, for which the mode amplitude does not vary along a field line (i.e., $k_\parallel = 0$), are usually stable in HSX. However, resistive ballooning modes [1] could be unstable in HSX if the modes are larger ("balloon") in regions that have bad local curvature (for the instability drive) and low local magnetic field shear (to minimize stabilizing effects of field line bending) within each field period — but with an envelope that extends over many field periods along field lines. Properties of possible resistive ballooning instabilities in HSX can be explored using the properties of resistive-interchange modes developed in (1)–(8) by making the adaptations of assuming $R_C^l$ is the local (superscript $l$) bad curvature radius and the local shear length $L_S^l$ represents an appropriate average of $1/L_S^2$ over the region along a field line where the mode amplitude is largest within a field period. Thus, a local instability parameter for resistive ballooning (RB) modes will be

$$D_{\mathrm{RB}}^l \equiv -\frac{(L_S^l)^2}{R_C^l} \frac{d\beta}{dr}, \quad \text{local resistive ballooning model instability parameter.} \tag{9}$$

This instability parameter will used in place of $D_I$ in estimating properties of resistive ballooning instabilities and the anomalous plasma transport they could induce in HSX.

In order to estimate instability parameters for resistive ballooning modes in HSX the following plasma parameters will be assumed [4] (at $r \sim 0.7a \simeq 0.07$ m): $T_e \simeq 100$ eV, $Z_{\mathrm{eff}} \sim 1.5$ (a guess), $n_e \simeq 10^{18}$ m$^{-3}$, $T_i \simeq 25$ eV, $\tau \equiv T_i/T_e = 0.25$, $B \simeq 0.5$ T, $L_p \simeq L_P \simeq 0.07$ m. These plasma parameters for HSX yield the following key parameters for resistive MHD modes: $D_\eta \simeq 1$ m$^2$/s, $\nu_\eta \simeq 200\,(k_y r)^2$/s, $c_A \simeq 10^7$m/s, $v_{Te} \simeq 6 \times 10^6$ m/s, $\varrho_S \simeq 2$ mm, $\nu_e \simeq 7 \times 10^4$/s, $\lambda_e \equiv v_{Te}/\nu_e \simeq 100$ m, $\beta \simeq 2 \times 10^{-4}$, $\omega_* \simeq 4 \times 10^4 (k_y r)$/s.

From the magnetic field properties that have been developed for HSX drift wave analyses [5], one can guess a local bad curvature length of $R_C^l \simeq 0.5$ m and a "typical" local magnetic shear length of $L_S^l \simeq 30$ m. Then, one obtains an effective $D_{\rm RB}^l \simeq 5$, $\omega_A \simeq 3 \times 10^5$/s, and $S \simeq 1500$. Assuming further that $k_y \simeq 1$/cm, one obtains $k_y \varrho_S \simeq 0.2$, $k_y r \simeq 7$, $\nu_\eta \simeq 10^4$, $\gamma_\eta \simeq 3 \times 10^5$/s, and $\delta_\eta \simeq 4$ mm, which would imply a mixing length fluctuation level (for a single $k_y$) of $\tilde{n}_{k_y}/n_e \sim \delta_\eta/L_n \sim 6\%$ and a turbulent diffusivity on the order of $D_{\rm turb} \sim 10$ m$^2$/s. These numbers are in reasonable agreement with observations of plasma fluctuations and anomalous transport at $r/a \simeq 0.7$ [4] in HSX: fluctuation decorrelation time $\gtrsim$ 3 $\mu$s and length $\sim 1$ cm, fluctuation level overall (i.e., summed over all $k_y$) $\gtrsim 10\%$, and anomalous diffusion coefficient $\sim 10$ m$^2$/s. Further, one can estimate an effective parallel wavenumber by $k_\| \simeq k_y \delta_\eta / L_S \simeq 1/(75\text{ m})$. This $k_\|$ yields $k_\|^2 v_{Te}^2/\gamma_\eta \nu_e \simeq 0.3 < 1$, which is consistent with a fluid response for electrons on the $1/k_\| \simeq 75$ m distance along field lines that the envelope of the resistive ballooning modes would extend over. Thus, if the assumptions that went into this analysis are reasonable, resistive-ballooning-type modes could be a candidate for explaining the turbulent plasma fluctuations and anomalous transport at $r/a \simeq 0.7$ in HSX.

There are, however, many important caveats and comments on this analysis:

1. *Effects due to* $\omega_*$. The diamagnetic flow frequency $\omega_*$ was neglected above. However, for the parameters used $\omega_* \simeq 4 \times 10^4 (k_y r) \simeq 2.8 \times 10^5$/s, which is almost equal to $\gamma_\eta$. Considering solutions of (1), which is a third order polynomial equation for $\omega$, $\omega_*$ effects become significant (for $\tau \equiv T_i/T_e << 1$) when $\gamma_\eta/\omega_* \lesssim (4/27)^{1/3} \simeq 0.5$; they substantially reduce the growth rate of the resistive ballooning modes, which propagate in the electron diamagnetic flow direction (for $T_i < T_e$), but do not completely stabilize them. Since $\gamma_\eta \propto (k_y r)^{2/3}$ whereas $\omega_* \propto k_y r$, at $r/a \simeq 0.7$ only modes with $k_y r \lesssim 7$ would be "robust" resistive ballooning modes with small $\omega_*$ effects. Fluctuations due to modes with $k_y r >> 7$ would have $k_\|^2 v_{Te}^2/\omega \nu_e > 1$ and thus would likely have adiabatic electron responses over all parallel scale lengths along field lines. Hence, $k_y r >> 7$ fluctuations at $r/a \simeq 0.7$ in HSX are likely to be due to drift wave instabilities.

2. *Effective local shear length* $L_S^l$. The key assumption in the preceding estimates is that the effective average local shear in the region along field lines where the resistive ballooning mode would be localized is characterized by $L_S^l \simeq 30$ m. While the global average magnetic shear $\hat{s} \equiv (r/q)(dq/dr)$ (with $q \equiv 2\pi/\iota$) is very small in HSX ($\hat{s} \lesssim 0.03$ for $r \lesssim 0.7a$), which would imply a global $L_S \simeq Rq/\hat{s} \gtrsim 30$ m, it is not clear what an appropriate value of $L_S^l$ is for resistive ballooning modes. Values of the $L_S^l$ greater than the 30 m used above would cause larger $k_y$ resistive ballooning modes to be unstable without significant $\omega_*$ effects because $D_{\rm RB}^l \propto (L_S^l)^2$. Smaller values give smaller $D_{\rm RB}^l$ and cause $\gamma_\eta$ to decrease relative to $\omega_*$, which would cause the range of $k_y r$ for robust instability of resistive ballooning modes to decrease (i.e., to less than the critical value of 7 estimated above).

3. *Perpendicular compressibility.* The preceding analysis neglected perpendicular compressibility effects on resistive ballooning modes introduced by an average over the combined effects of parallel variations of the geodesic curvature and mode amplitude along field lines [6]. The characteristic perpendicular compressibility frequency in tokamaks is $\omega_\perp = \sqrt{2}\, c_S/R_0$. It stabilizes very low $k_y$ resistive ballooning modes when $\gamma_\eta \lesssim \omega_\perp$. Using these tokamak-based formulas for resistive ballooning modes in HSX, one finds that only $k_y r \sim 1$ modes would likely be stabilized by perpendicular compressibility effects. But a full resistive ballooning mode analysis needs to be carried out for the HSX geometry to properly quantify these effects.

4. *Parallel extent of mode.* In the preceding analysis the parallel extent of the resistive ballooning mode was characterized by $1/k_\| \simeq 75$ m. This length is on the order of the electron collision length $\lambda_e \simeq 100$ m. It also represents going along field lines about 10 times around the toroidal circumference of HSX, or over about 40 of its field periods. Usually, one considers a fluidlike analysis to be appropriate only for $k_\| \lambda_e < 1$, which is violated by these numbers. However, since, as noted above, $k_\|^2 v_{Te}^2/\gamma_\eta \nu_e \sim 0.3 < 1$, the preceding fluidlike analysis might still be valid.

5. *Nearly adiabatic response within a field period.* While resistive ballooning mode analysis uses a fluid response for electrons on the long parallel length over which the mode extends, on the short scale length of one field period where $k_\|^l \sim N/R_0 = 4/R_0$, one obtains $(k_\|^l)^2 v_{Te}^2/\gamma_\eta \nu_e >> 1$ which implies an adiabatic electron response [7]. In the ballooning mode analysis the overall mode eigenfunction that would be observed experimentally is obtained by summing over the short-scale responses in each field period modulated by the long scale parallel envelope. It would thus would represent a combination of adiabatic and fluidlike responses. Thus, it might not be in disagreement with HSX measurements, which apparently show that to lowest order the electrons have a nearly adiabatic response with the density and potential fluctuations being nearly in phase.

6. *Electron inertia effects.* In the preceding estimates it was implicitly assumed that the electron collision frequency $\nu_e$ ($\sim 7 \times 10^4$/s) is larger than the mode growth rate $\gamma_\eta$ ($\sim 3 \times 10^5$/s). Since this is not the case, one should add electron inertia effects to the parallel Ohm's law used in the analysis in [2]. While this situation has apparently never been worked out in detail, its possible effects can be estimated by replacing the parallel electrical resistivity by $\eta_\| \to \eta_\|(1 + \gamma_\eta/\nu_e)$. This causes the mode width to scale as $\delta \sim D_I^{1/2} c/\omega_p \sim 1$ cm, and the mode growth rate to scale as $\gamma \sim \gamma_\eta (\gamma_\eta/\nu_e)^{1/2} \sim 2\gamma_\eta \sim 6 \times 10^5$/s. However, it would also need to be confirmed that this type of ballooning mode in which electron inertia determines the parallel mode length $1/k_\|$ ($\sim 35$ m here) is in fact unstable. If such modes are unstable, they might better be characterized as "electron inertia" ballooning modes rather than "resistive" ballooning modes.

7. *Perpendicular diffusion effects.* After a number of unstable resistive ballooning modes grow, develop a "bubbling" turbulent spectrum of modes, and cause anomalous diffusion, they introduce another characteristic damping frequency [3] $\gamma_D \sim k_x^2 D_{\rm turb}$. Using $k_x \sim 1/\delta_\eta$ and the HSX resistive ballooning mode estimates above one finds $\gamma_D \sim 6 \times 10^5$/s, which is a factor of 2 larger than the growth rate $\gamma_\eta$. Thus, maybe $D_{\rm turb}$ is a factor of 2 smaller, i.e., $D_{\rm turb} \sim 5$ m$^2$/s. However, a full, multi-mode nonlinear turbulence and turbulent transport simulation would be required to determine the appropriate value of $D_{\rm turb}$ for HSX plasmas.

8. **E**×**B** *flow shear stabilization?* For flow shear stabilization one would apparently need $\gamma_\eta < \omega_E \sim (k_y\delta)(E_r/B\,r) \simeq 30\,E_r$, or $E_r > 100$ V/cm, which seems a bit large and perhaps not experimentally feasible?

9. *Effects of parameter changes.* The main effect of changes in parameters in HSX is in the $k_y r$ boundary below which robust resistive ballooning instabilities might be present and above which drift wave instabilities should occur. Using a criterion $\gamma_\eta/\omega_* \sim 1$ to determine the boundary, one finds

$$(k_y r)_{\rm crit} \propto \frac{n_e B}{T_e^{5/2}} \frac{r L_P (L_S^l)^2}{(R_C^l)^2}. \tag{10}$$

   Thus, resistive ballooning instabilities should be restricted to a lower range of $k_y$ as one moves inward and $T_e$ increases. Conversely, if the density is increased, and after the magnetic field $B$ is doubled to 1 Tesla in upcoming HSX experiments, a broader range in $k_y$ should be unstable.

In summary, resistive (→ electron inertia?) ballooning modes may be a reasonable candidate for explaining the $k_y \lesssim 1$/cm fluctuations and the anomalous plasma transport they could induce at $r/a \sim 0.7$ in HSX. However, a lot of effects ($\omega_*$, effective magnetic shear length $L_S^l$, perpendicular compressibility, $k_\parallel \lambda_e$, electron inertia, turbulent spectrum and anomalous transport it induces) need to be explored before one can conclude these modes are in fact unstable and that they contribute significantly to the observed plasma fluctuations and anomalous transport over about the outer half of HSX plasmas.

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