Most Electron Heat Transport Is Not Anomalous; It's A Paleoclassical Process In Toroidal Plasmas

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(Dated: October 25, 2004)

Radial electron heat transport in low collisionality, magnetically-confined toroidal plasmas is shown to result from paleoclassical Coulomb collision processes (parallel electron heat conduction and magnetic field diffusion). In such plasmas the electron temperature is equilibrated along magnetic field lines a long length L (>> periodicity length $\pi R_0 q$), which is the minimum of the electron collision length and an effective field line length. Thus, diffusing field lines induce a radial electron heat diffusivity $M \equiv L/(\pi R_0 q) \sim 10 >> 1$ times the magnetic field diffusivity $\eta/\mu_0 \simeq \nu_e (c/\omega_p)^2$.

PACS numbers: 52.25.Fi, 52.35.Vd, 52.55.Dy, 52.55.Fa

For more than three decades the outstanding and pervasive mystery in pursuit of magnetic fusion has been [1–4]: what causes "radial" (across magnetic field lines) electron heat transport in magnetically-confined toroidal plasmas such as tokamaks. Because the experimentally-inferred electron heat transport exceeds the theoretical classical [5] (gyromotion-induced) and neoclassical [5] (drift-orbit-induced) collisional transport by factors of about 10^4 and 10^2, respectively, it is called "anomalous." Since this is often the dominant radial transport process, resolving this conundrum is very important both for understanding plasma confinement in present experiments and for developing accurate plasma performance predictions for the planned International Thermonuclear Experimental Reactor (ITER) [6].

Salient properties of radial electron heat transport observed in tokamak plasmas over the past three decades are [1–4]: 1) The experimentally-inferred effective radial electron heat diffusivity χ_e is of order 2 m^2/s (to within a factor of 10) in a wide variety of tokamak plasmas. 2) The inferred χ_e is typically 3–30 times the magnetic field diffusivity η/μ_0. 3) The χ_e usually increases from the hot plasma core toward the cooler edge. 4) Tokamak plasmas heat up to the "low collisionality," banana-plateau regime [5]. 5) In high density ohmically-heated plasmas χ_e is inversely proportional to the electron density (so-called Alcator scaling [7]). Some mysterious properties are: 6) The χ_e can be up to an order of magnitude smaller in the vicinity of low order rational surfaces [8], and "internal transport barriers" often form there [9, 10]. 7) The χ_e can also be smaller just inside a magnetic separatrix at the plasma edge [10]. This paper provides a new model for all these characteristics of radial electron heat transport based on the dominant Coulomb collision processes in low-collisionality toroidal plasmas.

The shortest time scale, most primitive and dominant Coulomb-collision-induced transport processes in magnetically-confined plasmas will be called paleoclassical processes. They occur on the electron collision time scale $1/\nu_e$. The dominant transport processes are parallel electron heat conduction and magnetic field diffusion. [Classical and neoclassical diffusion [5] develop on the same time scale but are smaller than the magnetic field diffusivity for most low electron pressure plasmas.]

On the $1/\nu_e$ time scale, electron heat conduction equilibrates the electron temperature over parallel (to the magnetic field $\mathbf{B}$) distances of order the electron collision length $\lambda_e \equiv v_{Te}/\nu_e$ in which $v_{Te} \equiv (2T_e/m_e)^{1/2}$. Magnetic field diffusion [see (7) below] is induced by the plasma resistivity η. It causes magnetic field lines to diffuse perpendicular to $\mathbf{B}$ with a diffusion coefficient $D_\eta \simeq \eta_0/\mu_0 \equiv \nu_e (c/\omega_p)^2 \sim (\Delta x)^2/\Delta t$, which implies a diffusive radial step $\Delta x \simeq \delta_e \equiv c/\omega_p$ [the electromagnetic (em) skin depth, in which $\omega_p \equiv (n_e e^2/m_e \epsilon_0)^{1/2}$ is the electron plasma frequency] in a collision time $\Delta t \simeq 1/\nu_e$. Thus, paleoclassical processes equilibrate T_e over a collision length λ_e (~ 200 m) along a field line that is diffusing radially about c/ω_p (~ 1 mm) — all in a collision time $1/\nu_e$ (typically ~ 10 μs). The parallel equilibration can be limited by the finite length of rational magnetic field lines in toroidal magnetic systems and by the parallel length over which field lines are diffusing.

In axisymmetric toroidal magnetic confinement systems the helical field lines form nested toroidal surfaces called magnetic flux surfaces. The "winding number" of field lines in tokamaks is defined by a "safety factor" (for kink stability) $q(r)$ in which r is the (cylindrical-like) radial label of the flux surface; it typically ranges from order unity in the plasma core to $\gtrsim 3$ at the edge. Flux surfaces are rational or irrational depending on whether or not q is the ratio of integers (m, n):

$$q(r) \begin{cases} = m/n, & \text{rational surface,} \\ \neq m/n, & \text{irrational surface.} \end{cases} \quad (1)$$

The irrational surfaces form a dense set while the rational surfaces are radially isolated from each other.

Rational surfaces are of interest because their helical magnetic field lines close on themselves after m toroidal (or n poloidal) transits. The length of such field lines on a $q(r_*) = q_* \equiv m/n$ rational surface in a large aspect ratio ($\epsilon \equiv r/R_0 << 1$) tokamak is $2\,\ell_* \simeq 2\pi R_0\, m \equiv 2\pi R_0 q_* n$, in which R_0 is the major radius of the torus.

Magnetic field lines diffuse for radial distances x off a rational surface greater than the em skin depth ($|x| > \delta_e$), but aren't created and don't diffuse for $|x| < \delta_e$ — see (7) below. In a sheared magnetic field the half-length ℓ_δ along $\mathbf{B}$ [see (6) below] over which field lines are diffusing is obtained [11] from $1 > k_x(\ell)\delta_e$ with $k_x = k_\theta \ell / L_S$, $k_\theta \equiv nq/r$ and $1/L_S \simeq rq'/R_0 q^2$: $\ell_\delta \equiv L_S/(k_\theta \delta_e)$. Setting $\ell_* = \ell_\delta$ determines a maximum n (typically $\gtrsim 10$) for field lines diffusing radially over their entire length:

$$n_{\max} = \frac{1}{\sqrt{\pi \delta_e q'}}, \;\; \delta_e \equiv \frac{c}{\omega_p}, \;\; q' \equiv \left|\frac{dq}{dr}\right|_{r_*}. \qquad (2)$$

The length of such rational helical field lines is

$$2\,\ell_{\max} \simeq 2\pi R_0 q_* n_{\max}, \;\text{ maximum field line length.} \quad (3)$$

However, in the vicinity of a low order rational surface $q^\circ \equiv m^\circ/n^\circ$ (e.g., 3/2), the relevant n is n° and the field line length is short ($2\,\ell_{n^\circ} \simeq 2\pi R_0 q^\circ n^\circ$).

The effective radial electron heat diffusivity χ_e can now be estimated phenomenologically. As a magnetic field line diffuses radially it carries with it the electron heat "contained" on the field line. The half-length L over which the electron temperature is equilibrated is

$$L = \min\{\,\ell_{\max},\, \lambda_e,\, \ell_{n^\circ}\}, \quad \text{equilibration length.} \quad (4)$$

Because L is longer than the poloidal periodicity half-length of magnetic field lines ($\pi R_0 q$), the paleoclassical (superscript pc) electron heat diffusivity is a multiple M larger than the magnetic field diffusivity in a torus:

$$\chi_e^{\rm pc} \sim M\frac{\eta}{\mu_0}, \quad M \equiv \frac{L}{\pi R_0 q}, \qquad (5)$$

which, except for constants, is this paper's main result.

To be more precise, the effects of paleoclassical processes in the vicinity of a medium order ($1 < n \leq n_{\max} \sim 10$) rational helical field line need to be quantified. Because the relevant properties of field lines are their poloidal and toroidal periodicities, the magnetic field curvature and torsion can be neglected. However, magnetic shear is important. Thus, a simple "sheared slab" model can be used to represent the magnetic field in the vicinity of a helical field line on a rational surface:

$$\mathbf{B} = B_0[\hat{\mathbf{e}}_{\rm z} + (x/L_S)\hat{\mathbf{e}}_{\rm y}] = B_0\hat{\mathbf{e}}_{\rm z} + \hat{\mathbf{e}}_{\rm z}\times\boldsymbol{\nabla}\psi. \qquad (6)$$

(A companion, more detailed paper [11] uses axisymmetric toroidal geometry magnetic flux coordinates.) Here, $B_0 =$ constant is the magnetic field strength, $\hat{\mathbf{e}}_{\rm z} \equiv \boldsymbol{\nabla} z$ is a unit vector along the rational field line, and $\psi \equiv B_0 x^2/2L_S$ is the magnetic flux function associated with the (small) magnetic shear. For a large aspect ratio tokamak the shear length is $L_S \simeq R_0 q/s$ with $s \equiv rq'/q$. Magnetic shear is caused by a (parallel) current flowing in the plasma: $\mathbf{J} = \boldsymbol{\nabla}\times\mathbf{B}/\mu_0 = \hat{\mathbf{e}}_{\rm z}\nabla^2\psi/\mu_0 = \hat{\mathbf{e}}_{\rm z} B_0/(\mu_0 L_S) \equiv J_z \hat{\mathbf{e}}_{\rm z}$. The preceding properties are evaluated on the rational surface where $x \equiv r - r_*$ vanishes.

The relevant magnetic field evolution equation can be obtained from Faraday's law together with a plasma Ohm's law of $\mathbf{E} = -\mathbf{V}\times\mathbf{B} + \eta\mathbf{J} + (m_e/n_e e^2)\, d\mathbf{J}/dt$, which includes electron inertia: $\partial\mathbf{B}/\partial t = -\boldsymbol{\nabla}\times\mathbf{E} = \boldsymbol{\nabla}\times(\mathbf{V}\times\mathbf{B}) - \boldsymbol{\nabla}\times(\eta\mathbf{J} + \mu_0\delta_e^2 d\mathbf{J}/dt)$ in which $d/dt \equiv \partial/\partial t + \mathbf{V}\boldsymbol{\cdot}\boldsymbol{\nabla}$. Using the pre-Maxwell Ampere's law $\boldsymbol{\nabla}\times\mathbf{B} = \mu_0\mathbf{J}$ and the fact that for the $\mathbf{B}$ in (6) $\partial\mathbf{B}/\partial t = -\boldsymbol{\nabla}\times(\partial\psi/\partial t)\hat{\mathbf{e}}_{\rm z}$, setting the coefficient of the $\hat{\mathbf{e}}_{\rm z}$ component inside the curl in the resultant equation to a constant yields the evolution equation for magnetic flux ψ:

$$\left(1 - \delta_e^2\nabla^2\right)\frac{d\psi}{dt} = \frac{\eta_\parallel}{\mu_0}\nabla^2\psi - \frac{\partial\Psi}{\partial t}. \qquad (7)$$

Here, the constant of the spatial integration is $\partial\Psi/\partial t = E_z^A(t)$, the inductive axial (toroidal) electric field, which is the "source" of magnetic flux ψ and hence field lines. This term represents the effect of the magnetic flux change in the central solenoid of a tokamak; it is negative so the Poynting flux is in the $-x$ direction — to balance, in equilibrium, the resistivity-induced magnetic flux diffusion [first term on the right in (7)] and thereby produce a stationary magnetic field on the resistive time scale. Resistive diffusion of the magnetic flux ψ is induced by the effect of the (parallel) plasma resistivity $\eta \to \eta_\parallel$ on the (parallel) current density $J_z \equiv (1/\mu_0)\nabla^2\psi$.

Since $\eta_\parallel/\mu_0 \sim \nu_e\delta_e^2$, for scale lengths x less than δ_e in (7) where the $\delta_e^2\nabla^2$ term dominates on the left: the magnetic field diffusivity becomes negligible, the solution for ψ becomes spatially constant [11], and hence no magnetic field lines are produced (i.e., $\hat{\mathbf{e}}_{\rm z}\times\boldsymbol{\nabla}\psi \to \mathbf{0}$) or diffuse in this region. The paleoclassical analysis will thus be restricted to $|x| > \delta_e$ and interpret that ψ represents diffusing field lines only for $|x| > \delta_e$, $|\ell| < \ell_\delta \leq \ell_{\max}$.

Neglecting the $\delta_e^2\nabla^2$ operator (assuming $x^2 > \delta_e^2$) in (7) and the advective $\mathbf{V}\boldsymbol{\cdot}\boldsymbol{\nabla}\psi$ term, which on the transport time scale ($t > 1/\nu_e$) is negligible [11] compared to the flux induced by D_η, the equation for $\psi(x,t)$ becomes a simple diffusion equation

$$\frac{\partial\psi}{\partial t} = D_\eta\frac{\partial^2\psi}{\partial x^2} - \frac{\partial\Psi}{\partial t}, \quad D_\eta \equiv \frac{\eta_\parallel}{\mu_0}. \qquad (8)$$

Since the time scales of interest are long compared to the electron gyrofrequency, the electron kinetics is usually described by a gyro-averaged kinetic equation which is called a drift-kinetic equation [5]. In the usual drift-kinetic equation magnetic flux surfaces and hence field lines are assumed to be stationary — but (8) indicates ψ obeys a diffusion equation. In particular, consider the evolution of the (small) magnetic flux (bundle of field lines) $\delta\psi(x,t)$ that penetrates the circular cross-section of the gyroorbit of an electron gyrating at its gyroradius $\varrho_e \equiv v_\perp/\omega_{ce} \sim 0.1$ mm around a magnetic field line. Substituting $\psi \to \psi_0 + \delta\psi(x,t)$ with

$\psi_0 \equiv B_0 x^2/2L_S$ into (8) and using the equilibrium relation $D_\eta\, \partial^2\psi_0/\partial x^2 \equiv \eta_\| J_z = E_z^A \equiv \partial\Psi/\partial t$, one obtains

$$\partial\, \delta\psi/\partial t = D_\eta\, \partial^2 \delta\psi/\partial x^2. \qquad (9)$$

Since the electron gyroradius is small ($\varrho_e << \delta_e$), $\delta\psi$ can be taken to be a unit delta function initially; for this initial condition the solution of (9) is

$$\delta\psi(x,t) = e^{-x^2/\,4D_\eta t}/\,(4\pi D_\eta t\,)^{1/2}; \quad t \geq 0. \qquad (10)$$

This small, initially localized flux diffuses radially with a radial spreading that grows linearly with time:

$$\langle x^2\rangle \equiv \int_{-\infty}^{\infty} dx\, x^2 \delta\psi(x,t) = 2D_\eta t. \qquad (11)$$

Thus, because of D_η, as time progresses a bundle of field lines initially located at $x = 0$ assumes a probability distribution given by (10) whose radial spread grows according to (11) — even for a stationary magnetic field!

This radial (x) diffusion of the bundle of magnetic field lines penetrating the electron gyroorbit causes the radial coordinate (x or ψ) of the electron guiding center to be a "stochastic" variable in the drift-kinetic equation. In particular, it implies (for $x^2 > \delta_e^2$, $\ell^2 < \ell_\delta^2 \leq \ell_{\max}^2$) a Fokker-Planck diffusion coefficient

$$\frac{\langle(\Delta x)^2\rangle}{\Delta t} \equiv \frac{d\langle x^2\rangle}{dt} = 2D_\eta, \qquad (12)$$

and a vanishing "drag" coefficient: $\langle\Delta x\rangle/\Delta t = 0$.

The "stochastic" field line diffusion effects are taken into account [12, 13] by adding a spatial Fokker-Planck diffusion operator to the usual drift-kinetic equation [5]:

$$\partial f/\partial t \,+\, v_\| \,\partial f/\partial \ell\big|_\psi = \mathcal{C}\{f\} + \mathcal{D}\{f\}. \qquad (13)$$

In this magnetic-field-diffusion-Modified Drift-Kinetic Equation (MDKE), $f(\mathbf{x},\mathbf{v},t) \to f(\psi,\ell,v_\|,v,t)$ is the distribution function, ψ is the radial field line label of the electron guiding center position, ℓ is the distance along a field line, $v_\| \equiv \mathbf{v}\cdot\mathbf{B}/B$ is the particle speed along $\mathbf{B}$, and $\mathcal{C}$ is the Coulomb collision operator. Particle drifts off field lines have been neglected because in magnetized axisymmetric toroidal plasmas their radial excursions ($\Delta x \sim \varrho\, q/\epsilon^{1/2}$) are small compared to the magnetic field diffusion scale length $\delta_e \equiv c/\omega_p$ for the usual low electron pressure situations where $\beta_e < \epsilon/q^2$. Finally, the spatial Fokker-Planck operator is [12, 13]

$$\mathcal{D}\{f\} \equiv \frac{\partial^2}{\partial x^2}\frac{\langle(\Delta x)^2\rangle}{2\Delta t} f = \frac{\partial^2}{\partial x^2} D_\eta f. \qquad (14)$$

The lowest order approximation to the MDKE (13) includes parallel free-streaming and Coulomb collisions:

$$v_\|\, \partial f_0/\partial\ell\big|_\psi = \mathcal{C}\{f_0\}. \qquad (15)$$

For long collision lengths λ_e compared to the (periodicity) length of a helical field line ℓ_*, its solution is a Maxwellian distribution constant along magnetic field lines, and hence a function of the local flux function ψ:

$$f_0(\psi,v,t) = n_e(\psi,t)\left(\frac{m_e}{2\pi T_e(\psi,t)}\right)^{3/2} e^{-m_e v^2/2T_e(\psi,t)}. \qquad (16)$$

Physically, the electron temperature T_e is equilibrated along helical field lines by parallel electron heat conduction. For $\lambda_e < \ell_*$ this process limits the parallel length over which this solution applies to λ_e. Thus, the Maxwellian f_0 in (16) is only applicable for $|\ell| \leq \ell_{f_M} \equiv \min\{\ell_*, \lambda_e\}$. The dependence of n_e and T_e on time t in (16) allows for their transport time scale evolution.

To obtain an electron energy balance equation one takes the energy ($\int d^3v\, m_e v^2/2$) moment of (13) using $f \simeq f_0$. However, since in toroidal geometry the length $2\,\ell_*$ of a helical rational field line is n times longer than the poloidal periodicity length $2\pi R_0 q$, to take account of the n times a helical field line wraps around the poloidal (periodicity) direction, the $\mathcal{D}\{f_0\}$ operator is also operated on by $\int_{-\ell_*}^{\ell_*} d\ell/2\pi R_0 q$. (Formally, in axisymmetric toroidal geometry this factor emerges [11] from a ballooning representation [11, 14] used to preserve the poloidal and helical periodicities for these "flute-like" responses in the vicinity of a rational surface.) Since T_e is only equilibrated over the parallel distance ℓ_{f_M}, and the maximum length of diffusing field lines is $\ell_{\max}$, this parallel integration is limited to $-L$ to L, where L is defined in (4) and the net effect is the multiplier M defined in (5). Assuming for simplicity that only an electron temperature gradient is present, one thus obtains:

$$\frac{3}{2}\frac{\partial T_e(x,t)}{\partial t} = \frac{\partial^2}{\partial x^2}\left[\chi_e^{\rm pc} T_e(x,t)\right] + \frac{Q_e}{n_e}, \quad \chi_e^{\rm pc} \equiv \frac{3}{2} M D_\eta, \qquad (17)$$

in which Q_e is the collisional electron heating. The relevant D_η and $\eta_\|$ (for a toroidal plasma) parallel (neoclassical, superscript nc) electrical resistivity are [11, 15]:

$$D_\eta \equiv \frac{\eta_\|^{\rm nc}}{\mu_0}, \quad \frac{\eta_\|^{\rm nc}}{\eta_0} \simeq \frac{\sqrt{2}+Z}{\sqrt{2}+13Z/4} + \frac{\mu_e}{\nu_e}. \qquad (18)$$

Here, Z ($\to$ $Z_{\rm eff}$ $\equiv$ $\sum_i n_i Z_i^2/n_e$) is the (effective) ion charge, the reference (perpendicular) resistivity is $\eta_0/\mu_0 \equiv m_e\nu_e/\mu_0 n_e e^2 \simeq 1.4\times 10^3 Z/[T_e({\rm eV})]^{3/2}$ m^2/s, and the parallel electron viscosity [15] $\mu_e/\nu_e = [Z+\sqrt{2}-\ln(1+\sqrt{2})]f_t/Zf_c \xrightarrow{Z=1} 1.5 f_t/f_c$ in the banana collisionality regime with $f_t \simeq 1.46\sqrt{\epsilon} + \mathcal{O}(\epsilon^{3/2})$ and $f_c \equiv 1-f_t$.

Equation (17) is a diffusion-type equation for the electron temperature T_e with a paleoclassical diffusion coefficient $\chi_e^{\rm pc}$. Since $\chi_e^{\rm pc} \sim MD_\eta$, T_e relaxes a factor of order M faster than the flux ψ does [cf., (8) and (17)].

In typical toroidal plasmas where $\lambda_e > \ell_{\max}$, $\chi_e^{\rm pc} = (3/2)\, n_{\max}\, \eta_\|^{\rm nc}/\mu_0$. For shorter λ_e or near a low order rational surface, the parallel equilibration length is limited by these effects [11], as indicated in (4). Because the distance between a low order rational surface $q^\circ \equiv m^\circ/n^\circ$ and the nearest rational surface with $n = n_{\max}$ is $\delta x^\circ \simeq 1/(n^\circ n_{\max} q') = (\pi\delta_e/q')^{1/2}/n^\circ$ [or, at a minimum in q, $\delta x^\circ_{\min} \simeq (2/n^\circ)^{2/3}(\pi\delta_e/q'')^{1/3}$], only the lowest n° rational surfaces (e.g., 1/1, 3/2, 2/1) will be isolated enough radially from other $n \leq n_{\max}$ rational surfaces for $\chi_e^{\rm pc}$ to be small ($\sim n^\circ\eta/\mu_0$) in their vicinity.

The salient and mysterious properties of "anomalous" radial electron heat transport identified in the second paragraph can be interpreted in terms of the paleoclassical model developed in (4), (5), (17) and (18) as follows: 1) Magnitude. For a typical ohmically-heated TFTR plasma [16] $T_e \simeq 1.2$ keV, $n_e \simeq 3\times 10^{19}$ m^{-3}, $Z_{\rm eff} \simeq 2$, $R_0 \simeq 2.5$ m, $q \simeq 1.6$, and $1/q' \simeq 0.4$ m at the plasma half-radius ($r/a \simeq 0.4/0.8 = 0.5$), which yields $\eta_0/\mu_0 \simeq 0.067$ m^2/s, $\eta_\parallel^{\rm nc}/\eta_0 \simeq 2.2$, $c/\omega_p \simeq 10^{-3}$ m, $n_{\max} \simeq 11$, and $\lambda_e \simeq 300$ m $> \pi R_0 q\, n_{\max} \simeq 140$ m, so that $L = \pi R_0 q\, n_{\max}$, $M = n_{\max} \simeq 11$, and the estimated $\chi_e^{\rm pc}$ is 2.5 m^2/s $\sim \chi_e^{\rm exp}$. Since this $\chi_e^{\rm pc} \propto T_e^{-3/2}$, it becomes less than 1 m^2/s for $T_e \gtrsim 2$ keV — and it may then be smaller than possible microturbulence-induced transport. 2) Ratio To D_η. For this TFTR case one has $\chi_e^{\rm pc}/D_\eta = (3/2)M \simeq 17 >> 1$. 3) Radial Variation. In the usually applicable "collisionless paleoclassical regime" ($\lambda_e > \pi R_0 q\, n_{\max}$), $\chi_e^{\rm pc} \propto T_e^{-3/2}$ increases as T_e decreases from the hot plasma core toward the cooler edge. 4) Collisionality Regime. Tokamak plasmas will ohmically heat until T_e is limited by $\chi_e^{\rm pc}$, which is relevant only if $\lambda_e > \pi R_0 q$ (banana-plateau collisionality regime [5]). 5) Density Scaling. For high density "collisional" plasmas with $\pi R_0 q < \lambda_e < \pi R_0 q\, n_{\max}$, $\chi_e^{\rm pc} \propto (v_{Te}/R_0 q)(c/\omega_p)^2 \propto 1/n_e$, and $\tau_{Ee} \simeq a^2/4\chi_e^{\rm pc} \simeq 0.27\,(n_e/10^{20}{\rm m}^{-3})\,a^2 R_0 q$ [asssuming $T_e^{1/2} \simeq (500\ {\rm eV})^{1/2}$], which is an Alcator-like energy confinement scaling law [7]. 6) Low Order Rational Surfaces. As indicated in (4), L and hence $\chi_e^{\rm pc}$ are much smaller near low order ($n^\circ = 1, 2$) rational surfaces, particularly when q is near a minimum. 7) Near Separatrix. On closed field lines just inside a magnetic separatrix, q and q' are large, and $n_{\max}$ and $\chi_e^{\rm pc}$ are reduced [11].

Perhaps the most remarkable paleoclasssical predictions are the reduced χ_e's near low order rational surfaces, in agreement with some key experimental results. For example, RTP experiments [8] showed that as highly localized electron cyclotron heating (ECH) was moved radially outward the central T_e had a "stair-step" behavior — it decreased abruptly as the ECH passed each low order rational surface, which indicated low χ_e at such surfaces. Also, jumps in T_e (over a narrow radial region approximately predicted by $2\,\delta x^\circ_{\min}$) have been observed in evolving DIII-D L-mode plasmas [17] as an off-axis minimum in $q(r)$ passes low order rational surfaces. Finally, the paleoclassical model predicts a $\chi_e^{\rm pc}$ profile, magnitude, and barrier width in reasonable agreement with JT-60U experiments [9] that used reversed central shear ($q' < 0$) and apparently $q^\circ_{\min} = 3/1$ to produce a large T_e gradient in an electron internal transport barrier.

The author thanks his colleagues at University of Wisconsin-Madison, especially C.C. Hegna, V.V. Mirnov, S.C. Prager, K.C. Shaing and C.R. Sovinec, and P.B. Parks and R.E. Waltz of General Atomics, for their insightful questions and comments on this work. He is also grateful to the U.S. Department of Energy for support of his research over the past three decades.

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